World Science Scholars

COURSE 1

A BEAUTIFUL UNIVERSE:

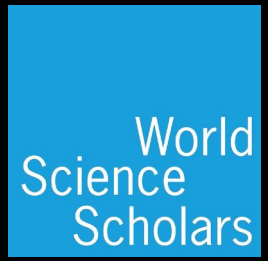
Black Holes, String Theory, and the Laws of Nature as Mathematical Puzzles

CUMRUN VAFA, PHD ■ HARVARD UNIVERSITY

MODULE 1

A BEAUTIFUL UNIVERSE:

Black Holes, String Theory,
and the Laws of Nature as Mathematical Puzzles



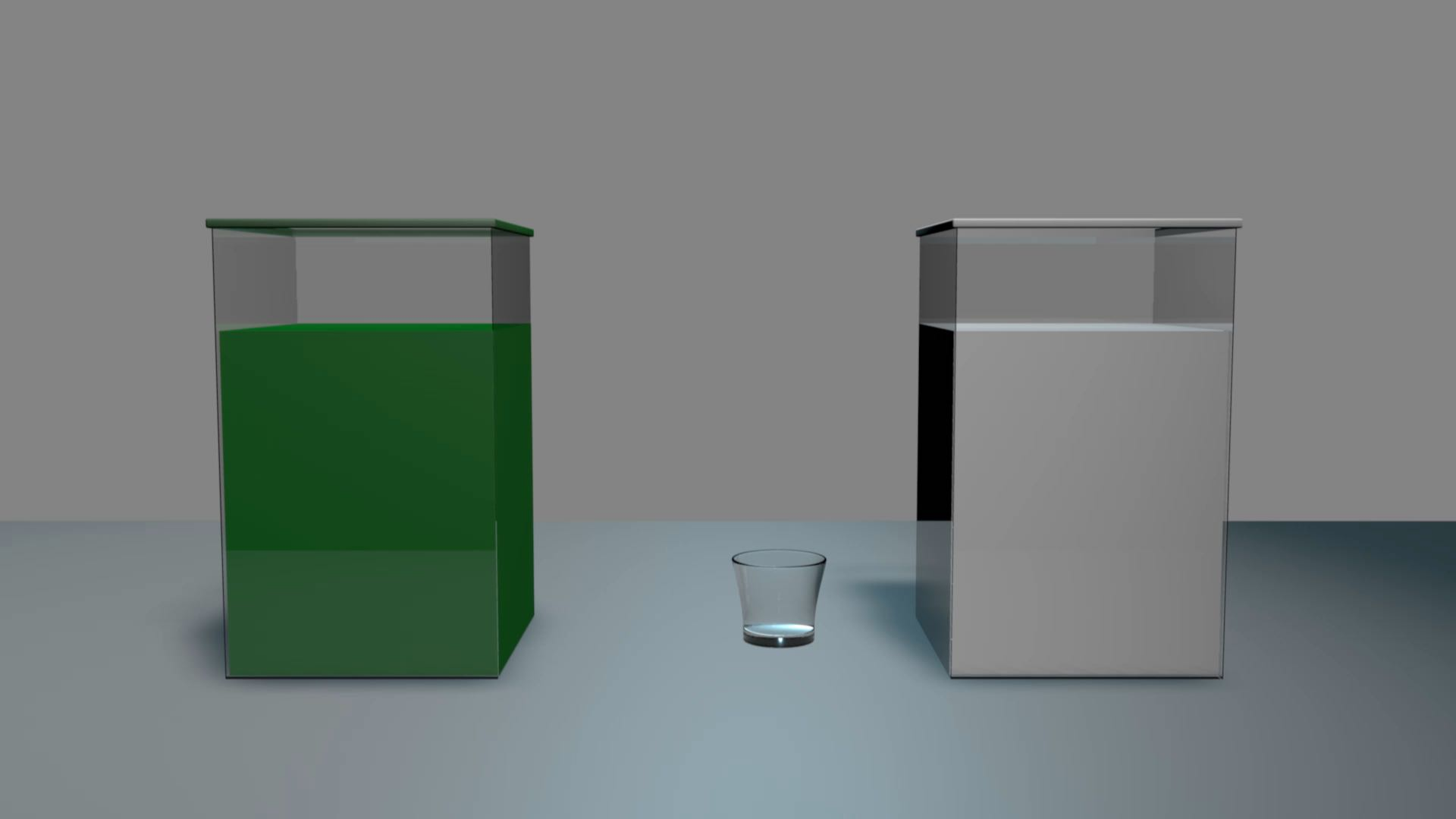
Cumrun Vafa, PhD
Harvard University

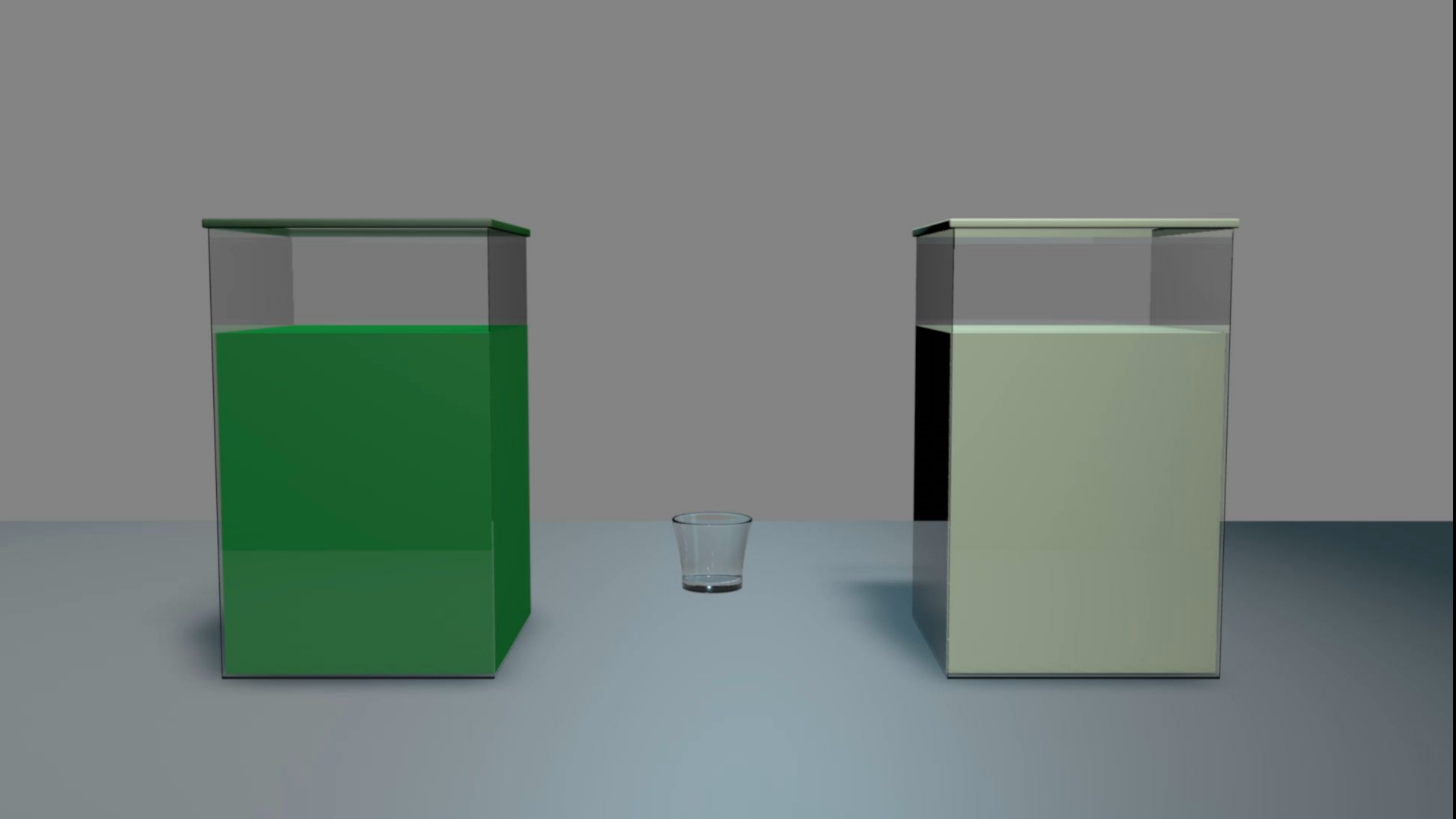
MATH & PHYSICS CONNECTIONS

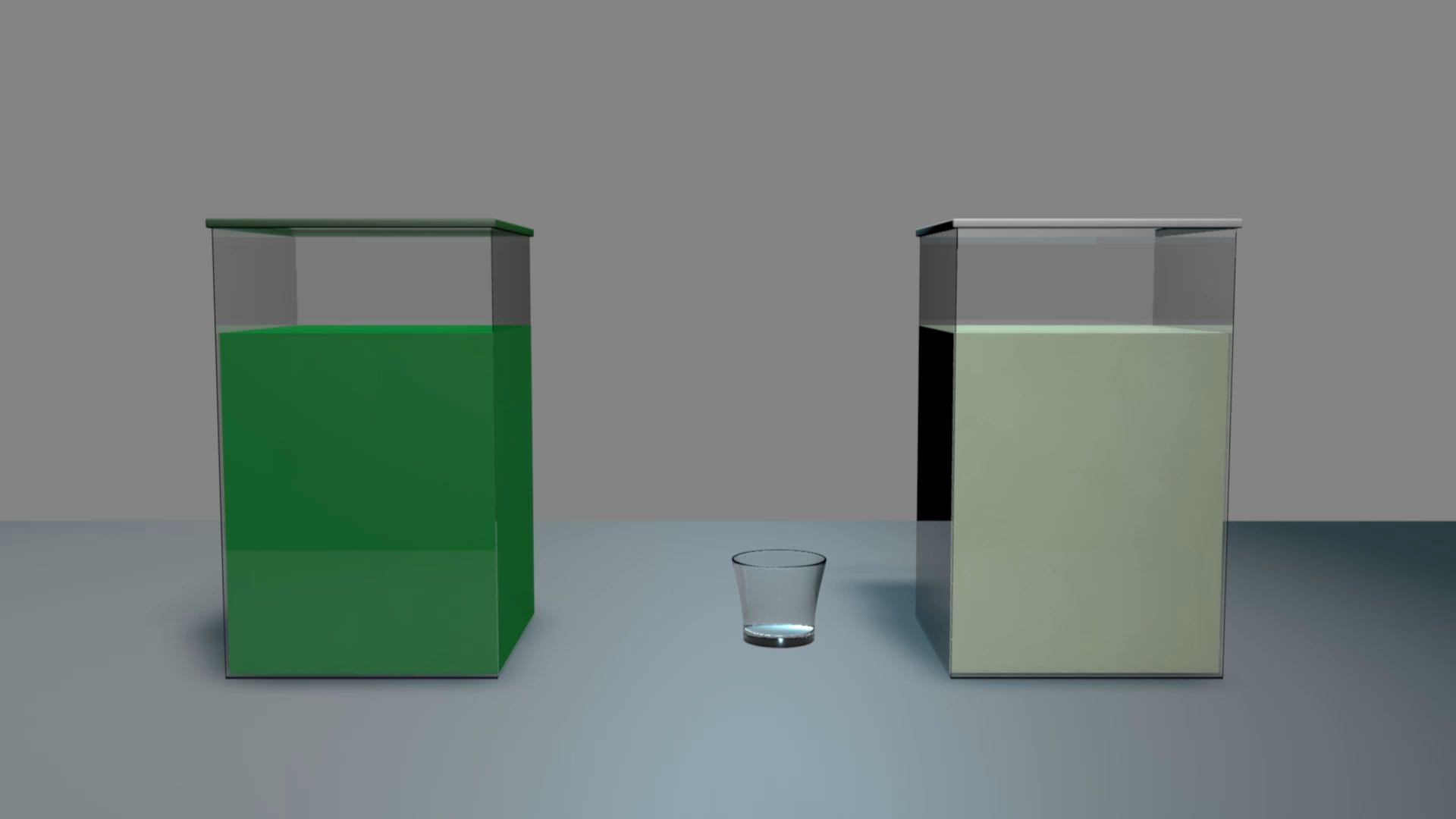
Deep physical ideas have simple mathematical underpinnings.

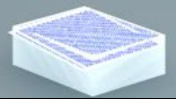
We will explore puzzles that illuminate those math-physics connections.

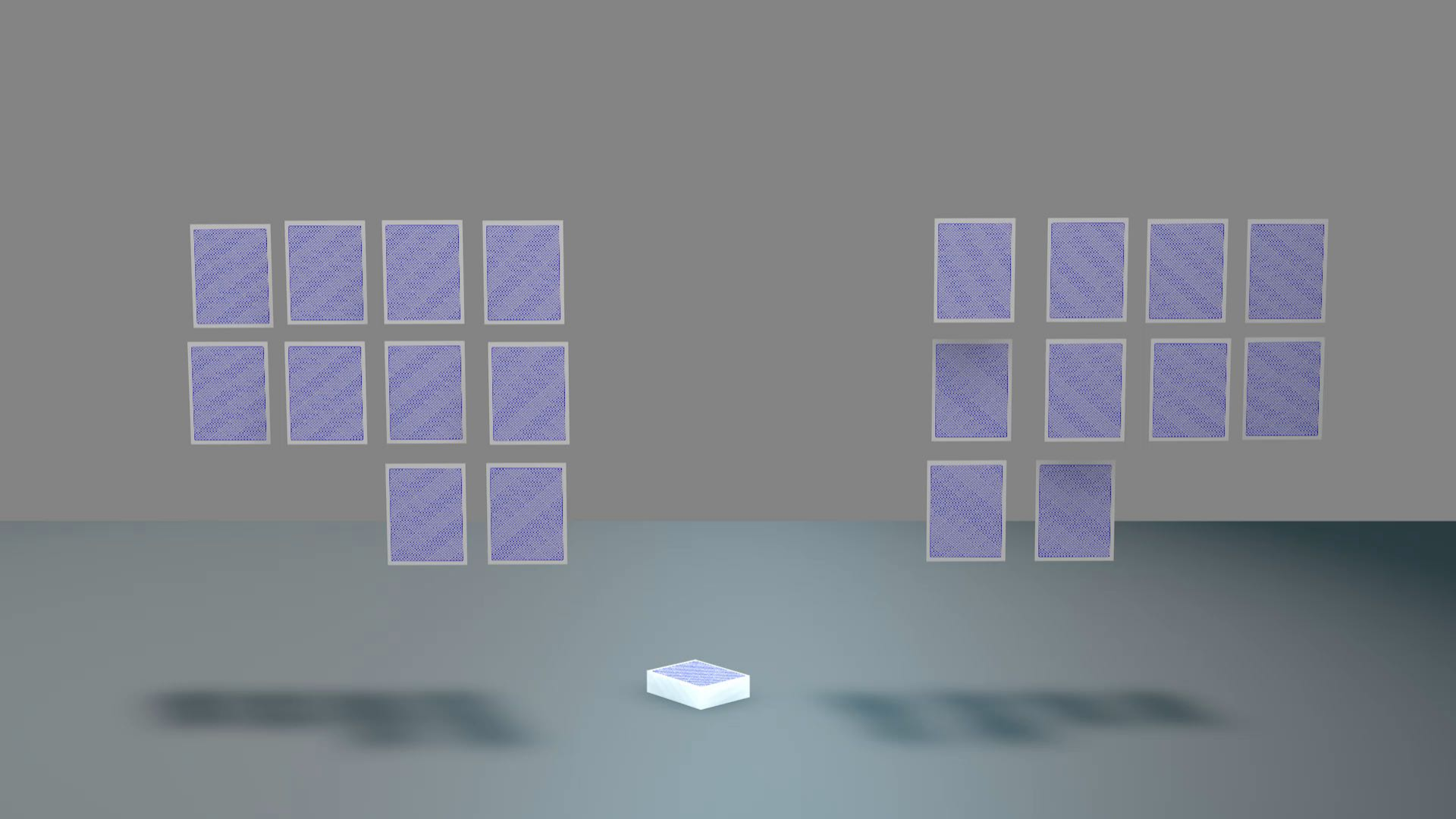
PUZZLE 1: MIXING PAINTS









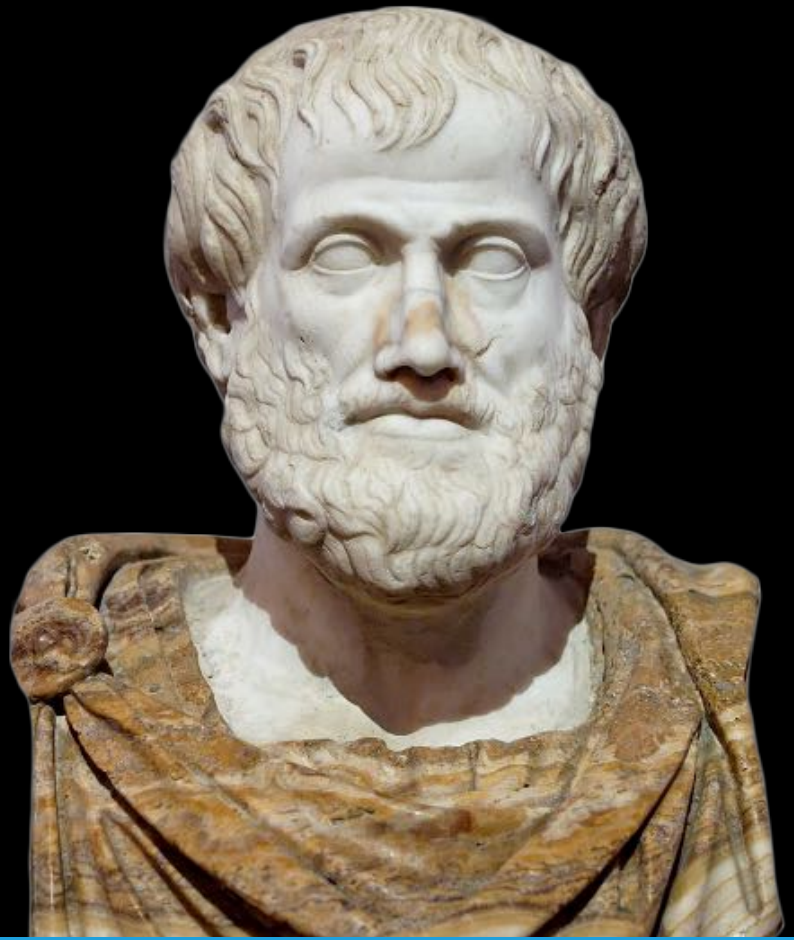


PUZZLE 1

Math and Physics Connections:
Symmetry and Conservation Laws

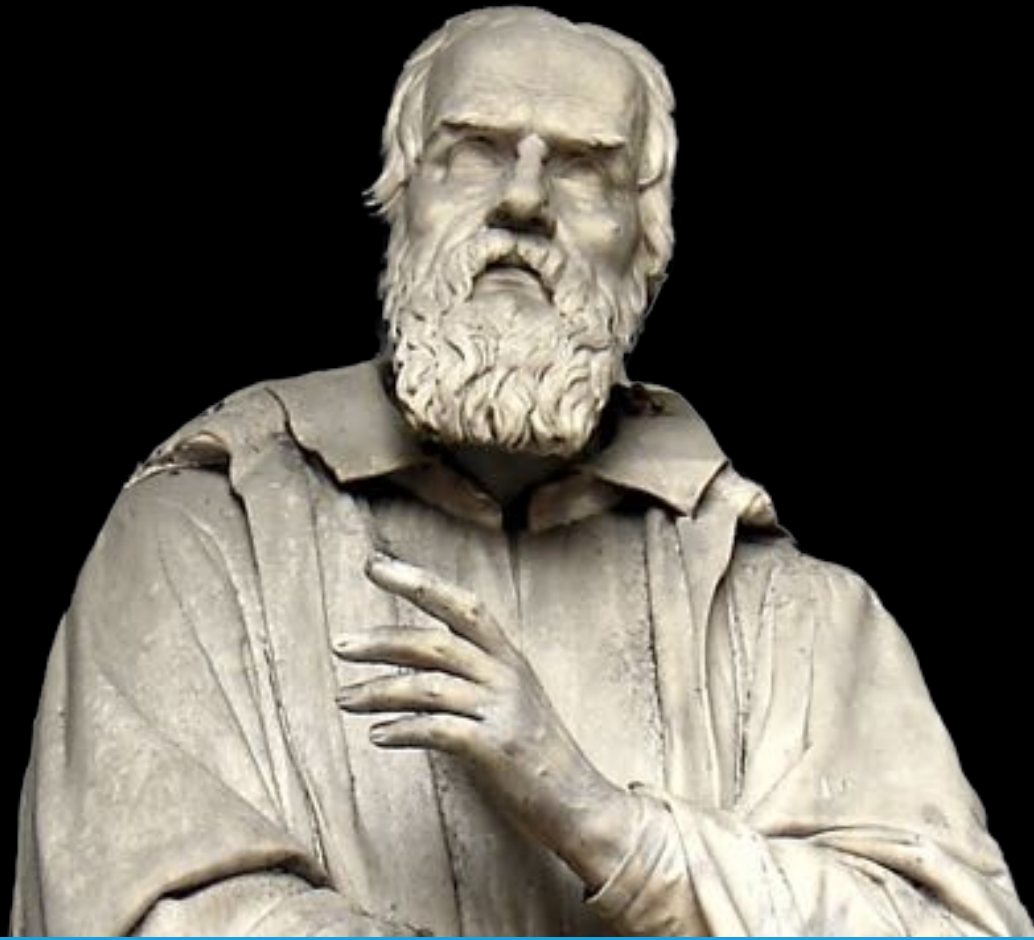
ARISTOTLE

Heavier objects fall faster



GALILEO

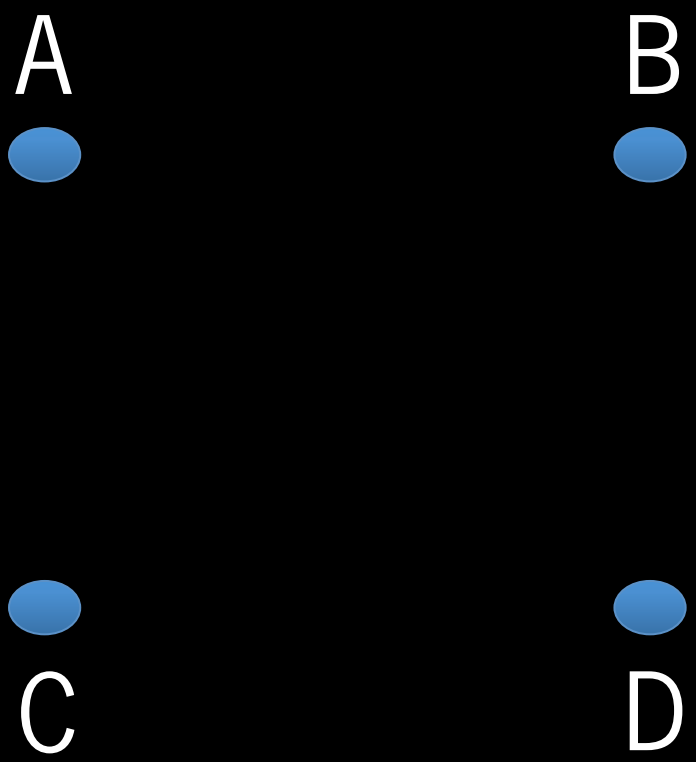
All objects fall at the same rate

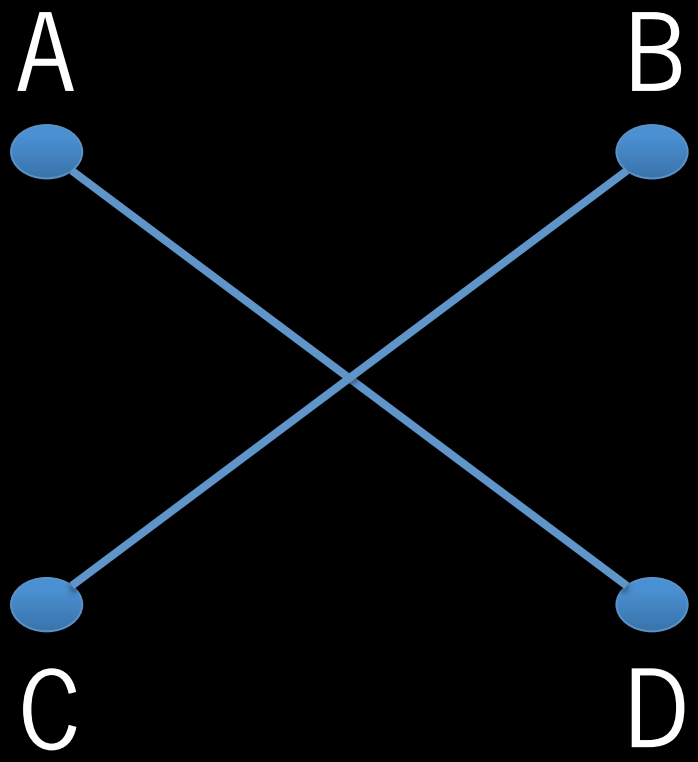


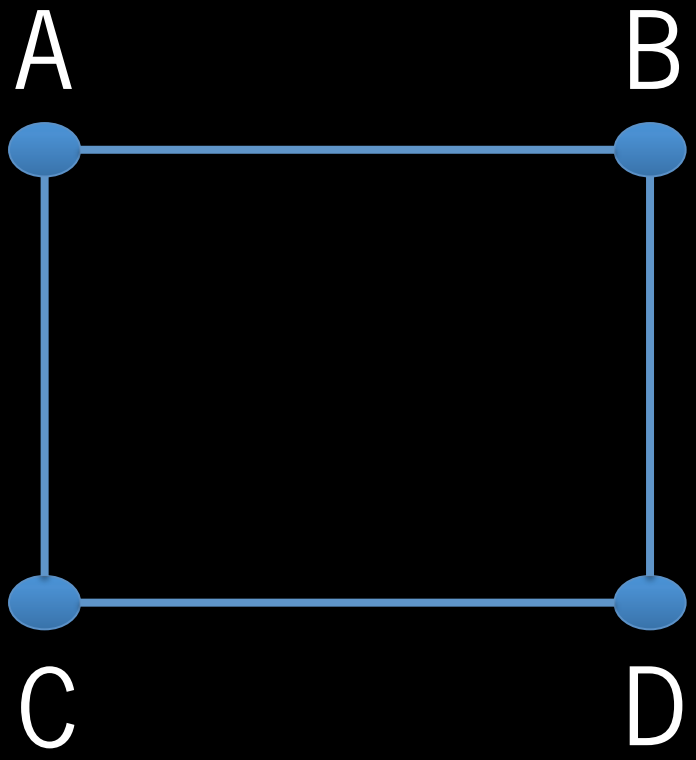
PUZZLE 1

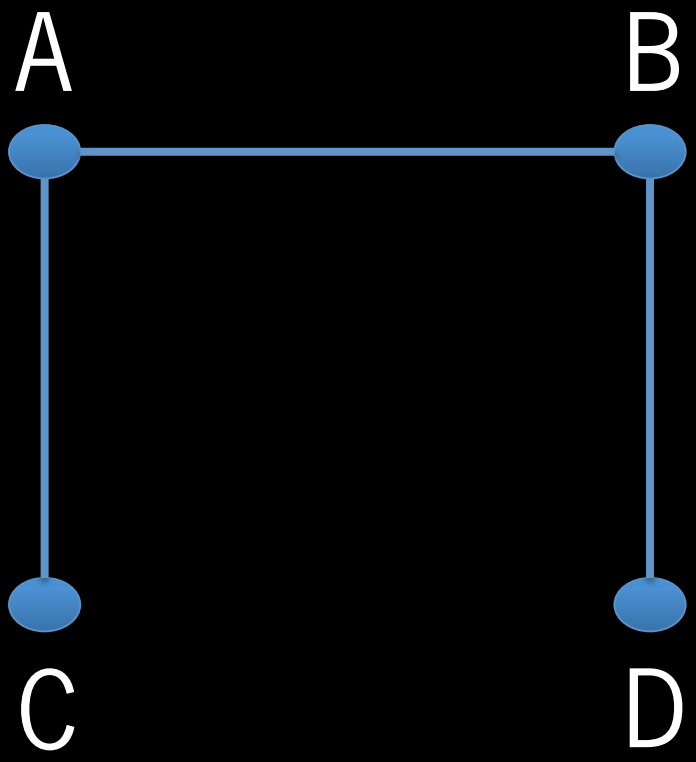
Math and Physics Connections:
Symmetry and Conservation Laws

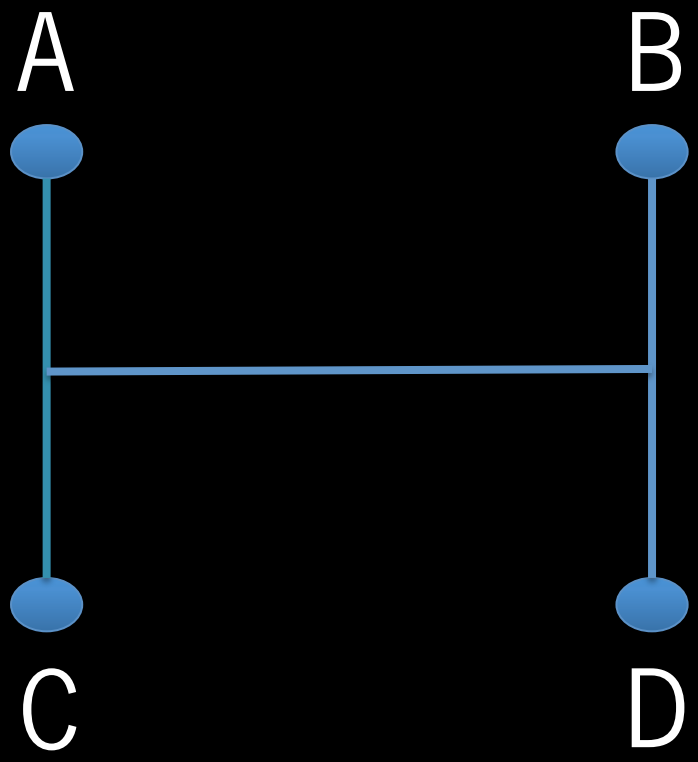
PUZZLE 2: DESIGNING A HIGHWAY SYSTEM

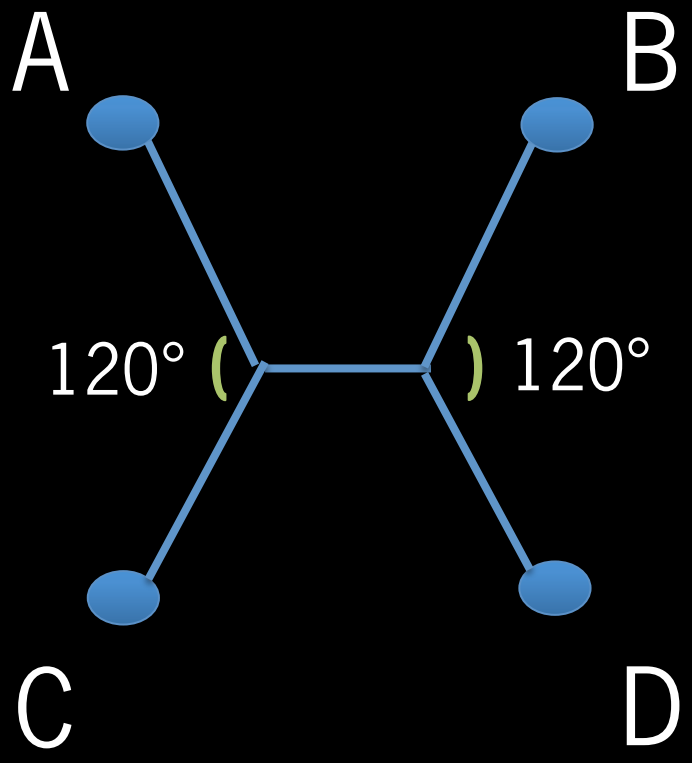

A
B
C
D


A
B
C
D


A
B
C
D


A
B
C
D


A
B
C
D


A
B
120°
120°
C
D

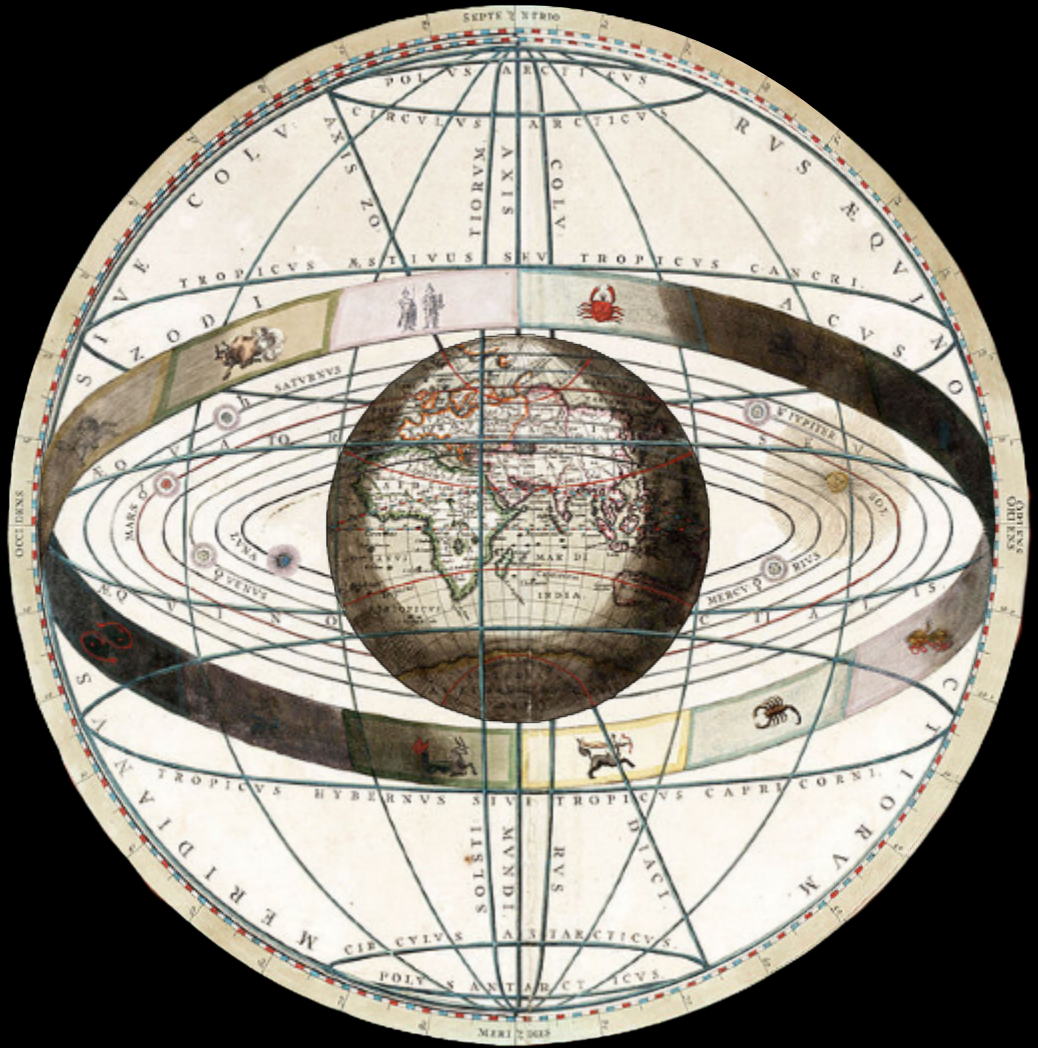
PUZZLE 2

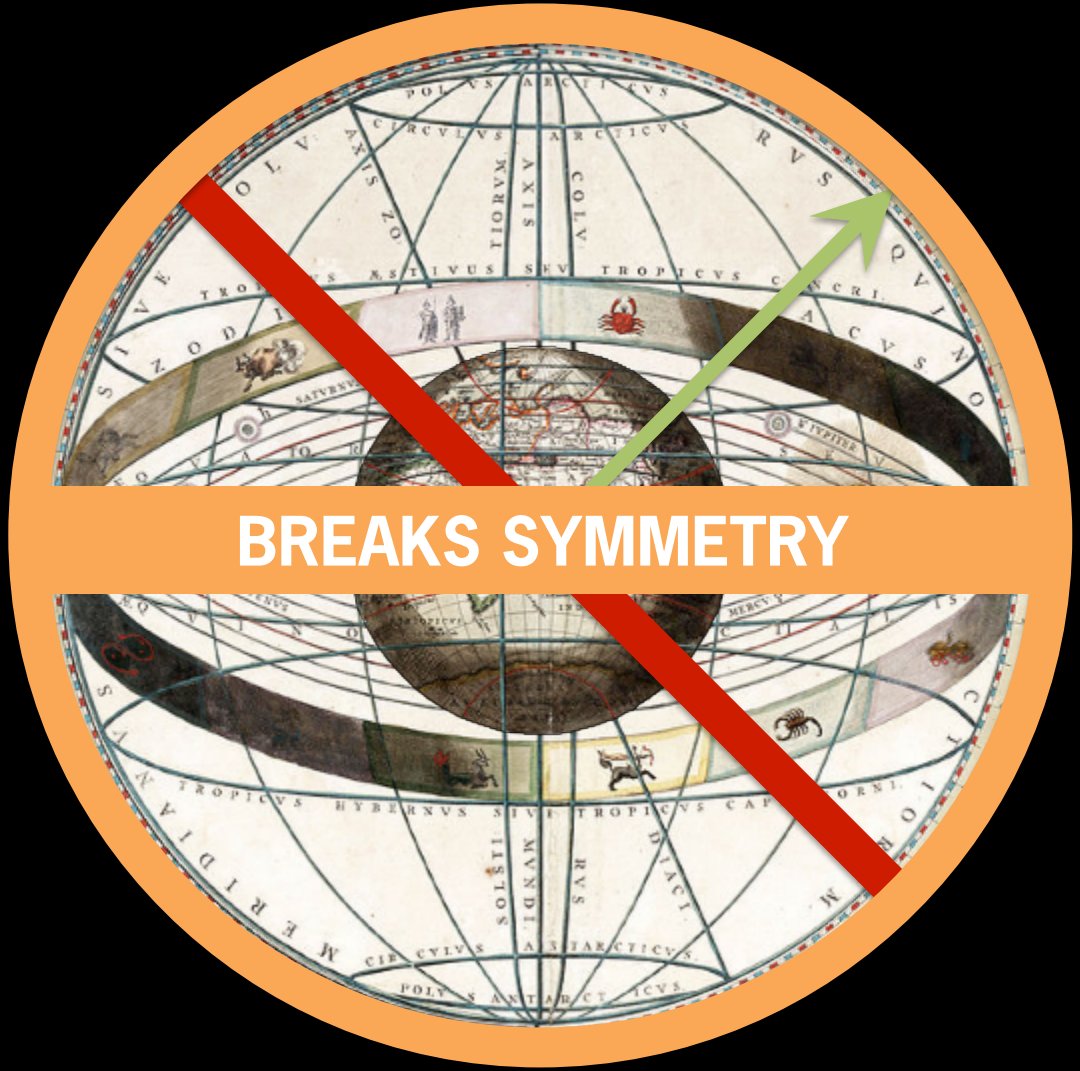
Math and Physics Connections:
Symmetry and Symmetry Breaking

EARLY GREEK PHILOSOPHERS

Earth is round and at the center of the universe

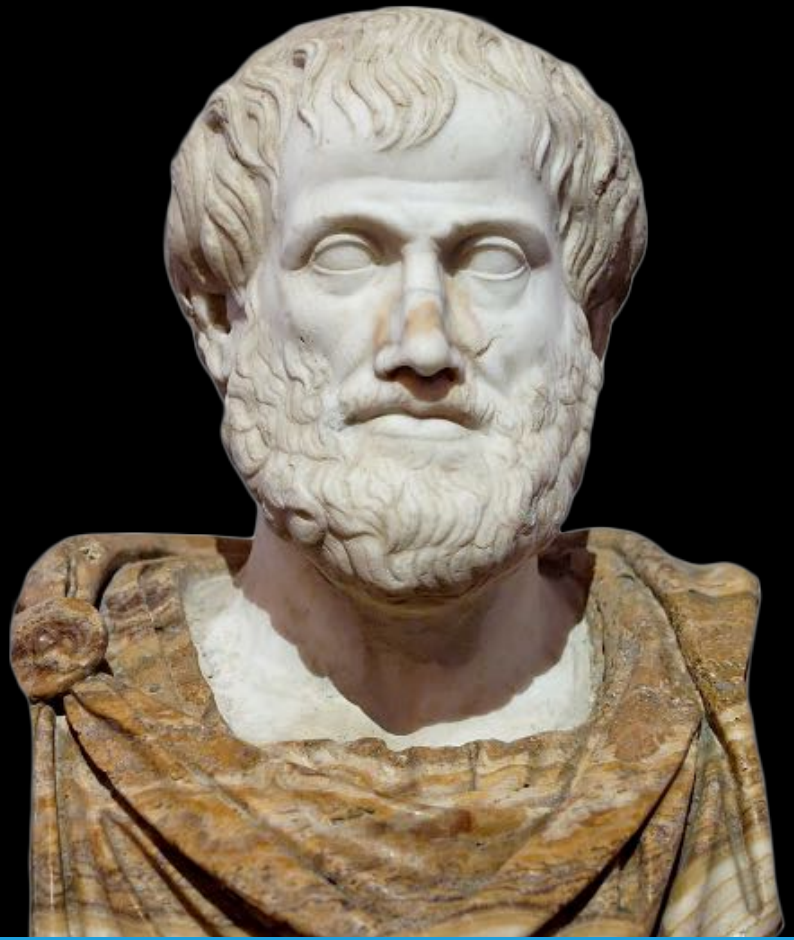
And it is not moving!

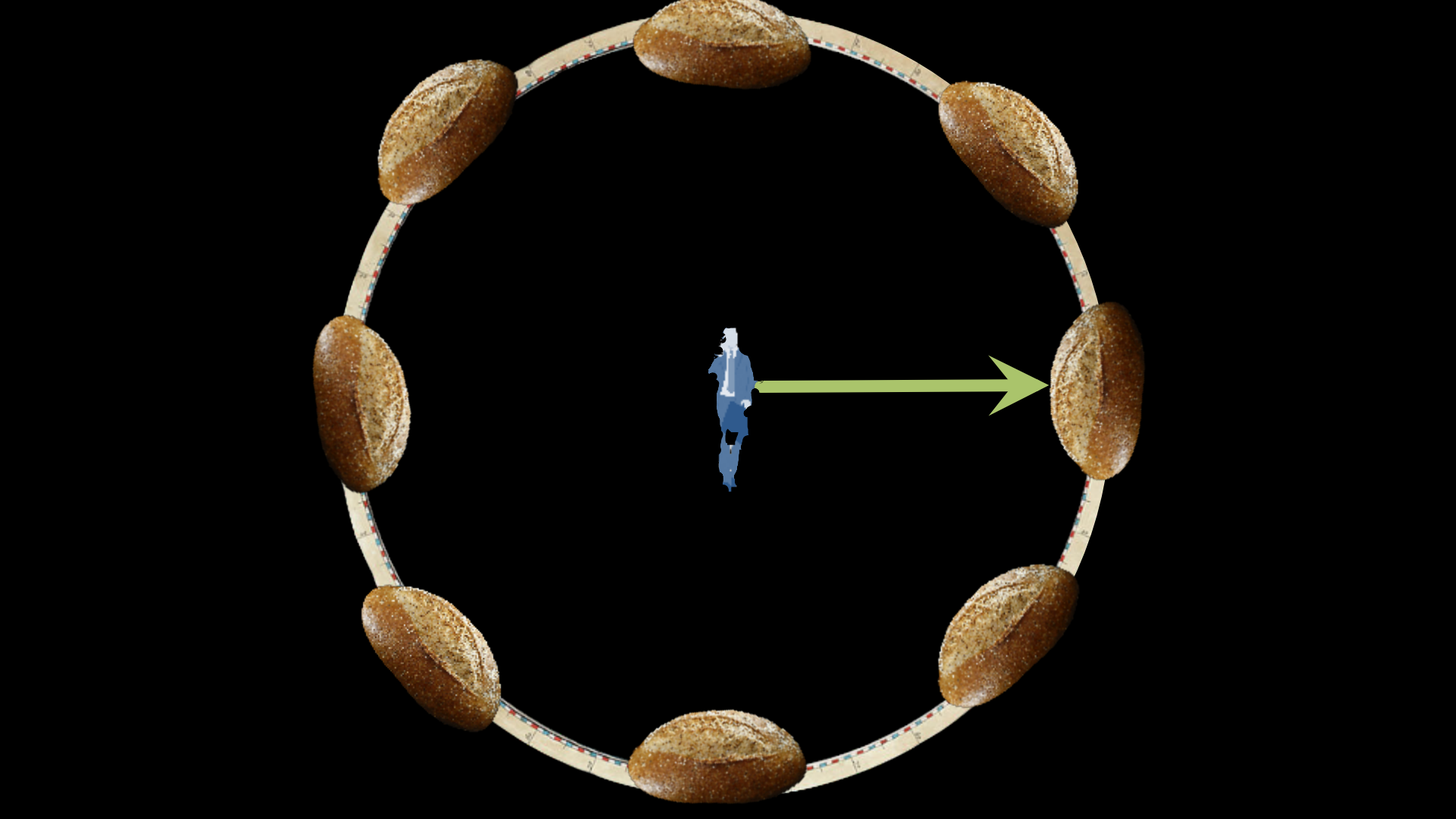

SEPTENTRIO
POLVS ARCTICVS
CIRCVLVS ARCTICVS
AXIS ZODIACORVM SIVE SOLSTITIORVM
COLVRVS
TROPICVS ÆSTIVVS SIVE TROPICVS CANCRI
ZODIACVS
SATVRNVS
IVPITER
SOL
MARS
LVNA
VENVS
MERCVRIVS
MAR DI
INDIA
TROPICVS HYBERNVS SIVE TROPICVS CAPRICORNI
MERIDIANVS CIRCVLVS
POLVS ANTARCTICVS
ORIENS
OCCIDENS
MERIDIES


BREAKS SYMMETRY

ARISTOTLE

“Not a good argument!”





Spontaneous symmetry
breaking is imprinted on our bodies!

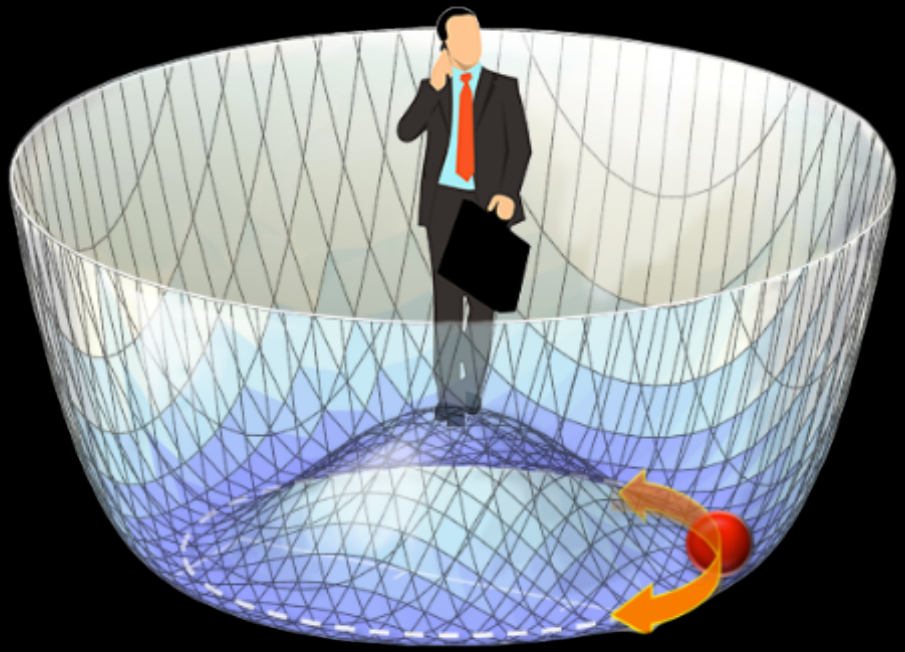
PUZZLE 2

Math and Physics Connections:
Symmetry and Symmetry Breaking

Why is symmetry breaking
important in physics?

MODERN APPLICATION OF SYMMETRY BREAKING

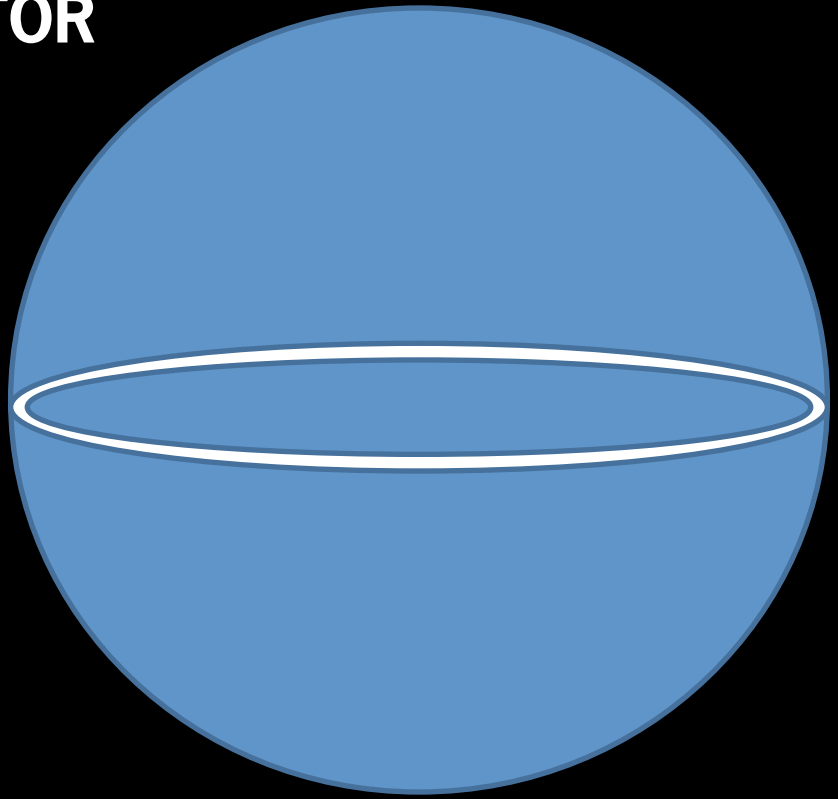
Higgs particle and
the origin of mass

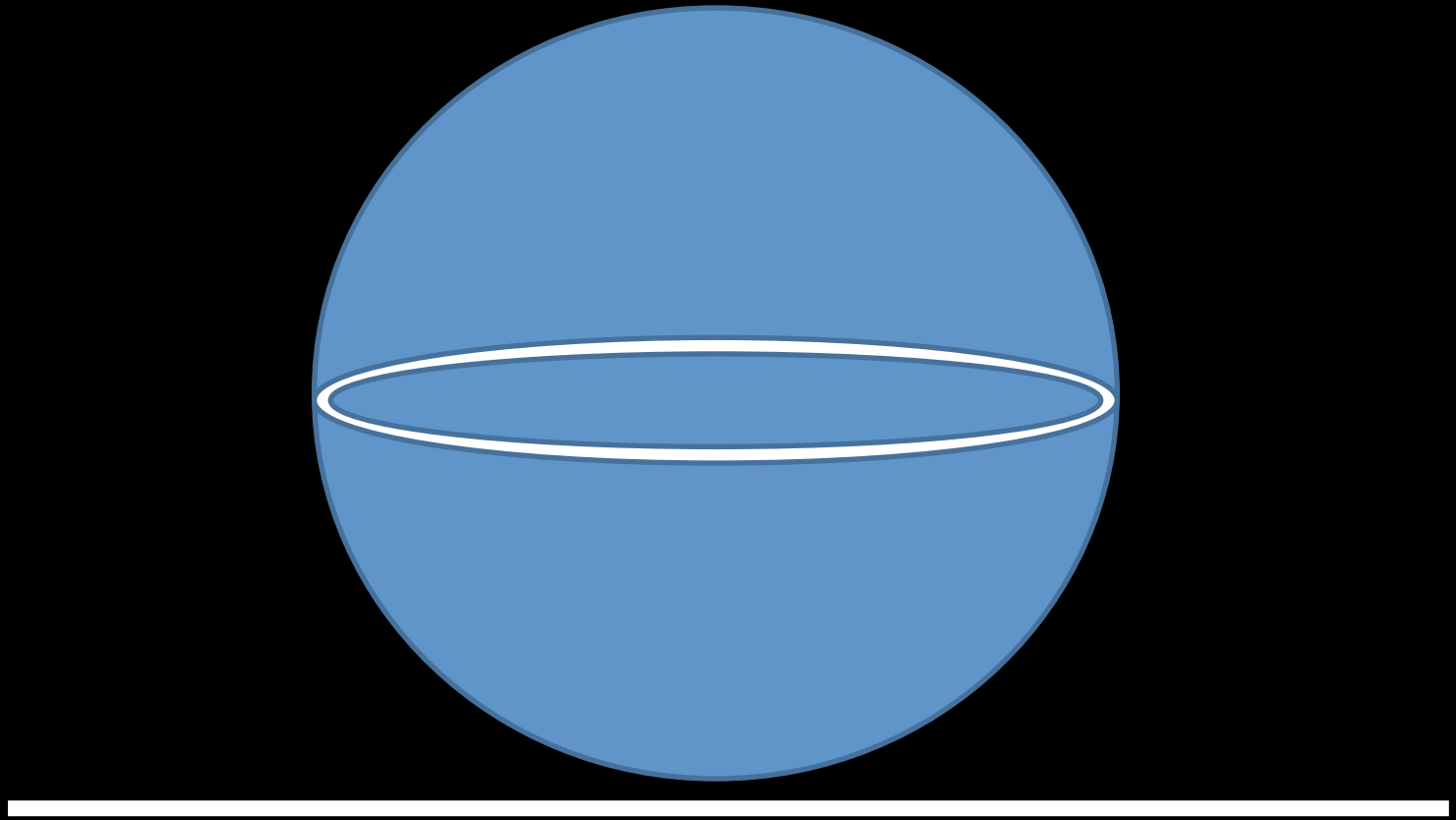


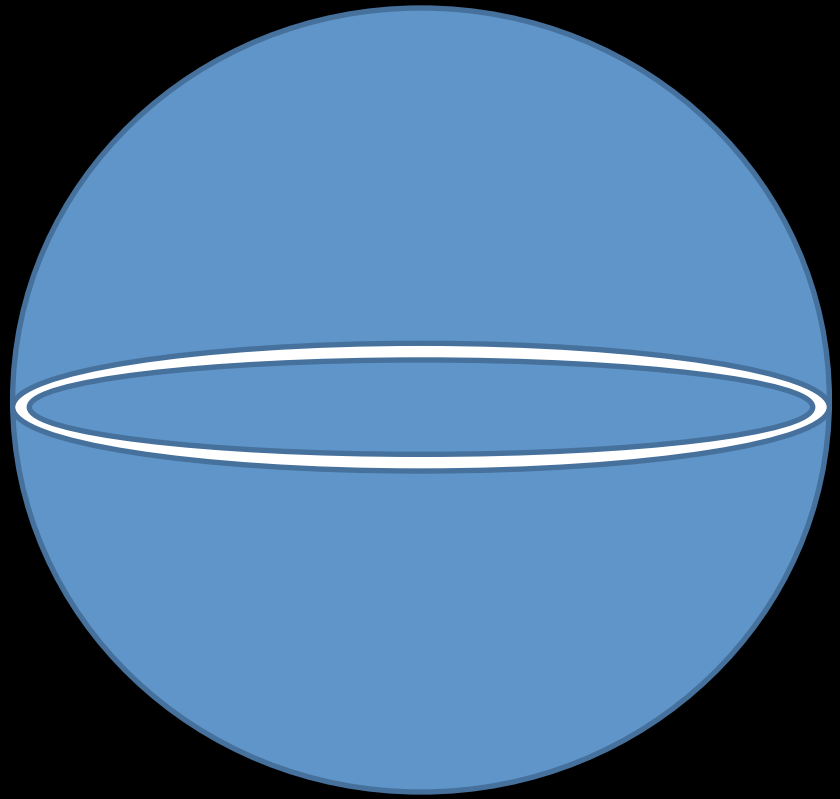
PUZZLE 3

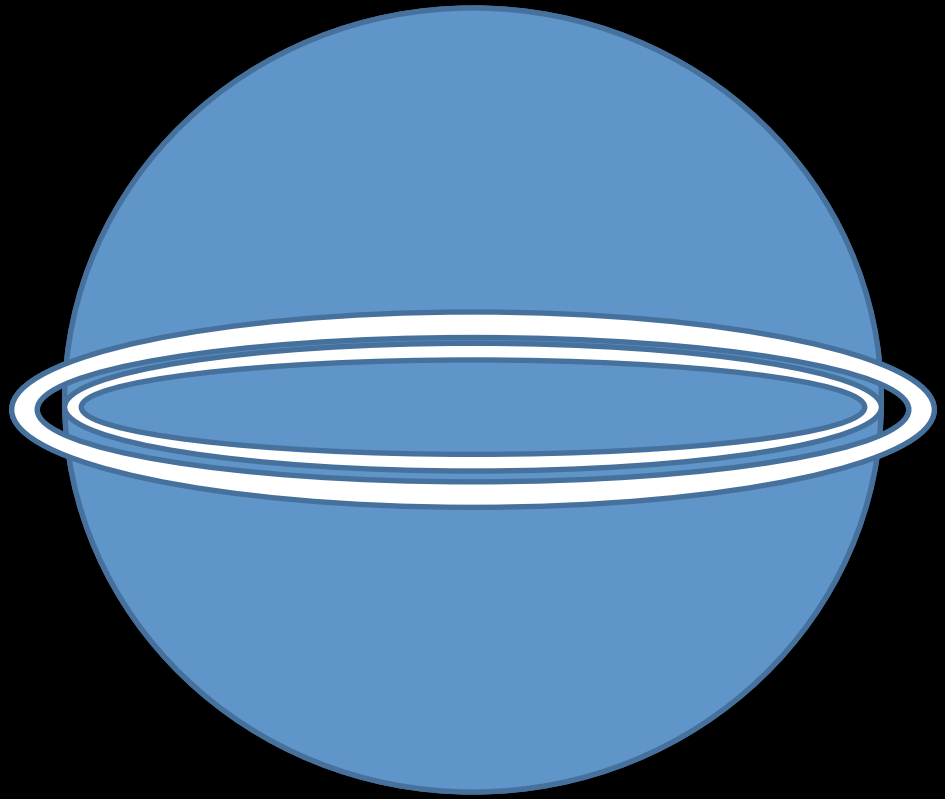
Unreasonable Power of Simple Mathematics

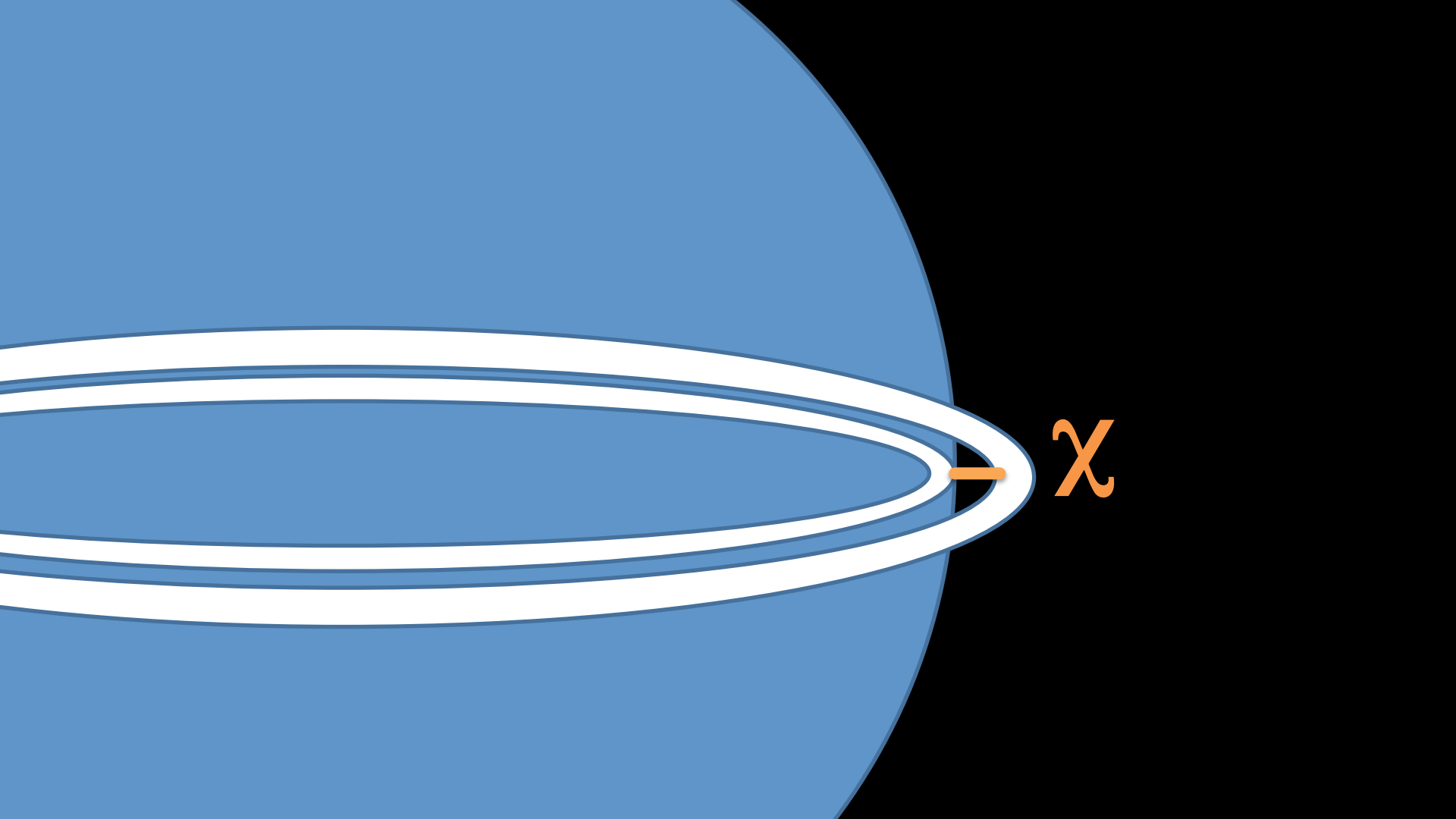
EARTH AND THE EQUATOR





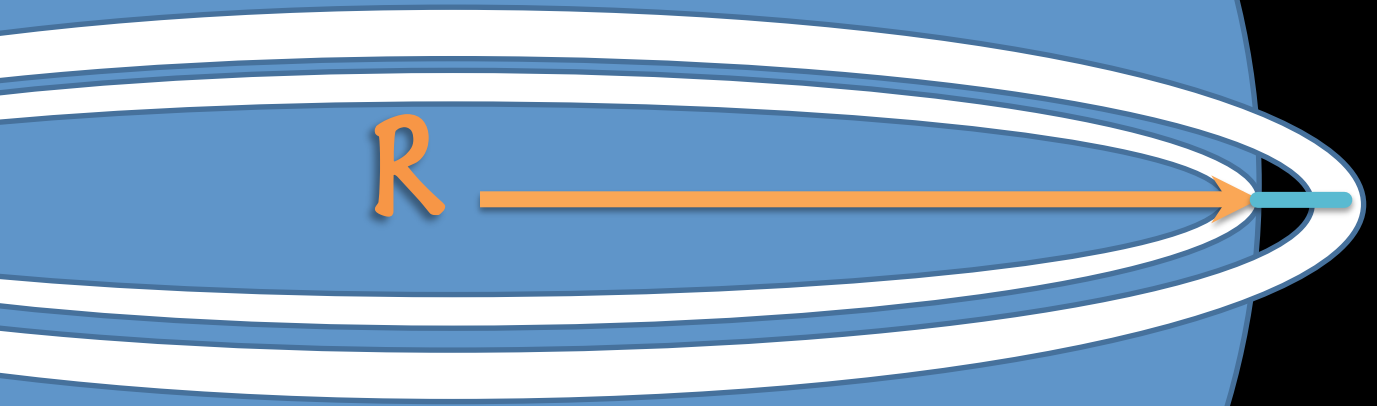


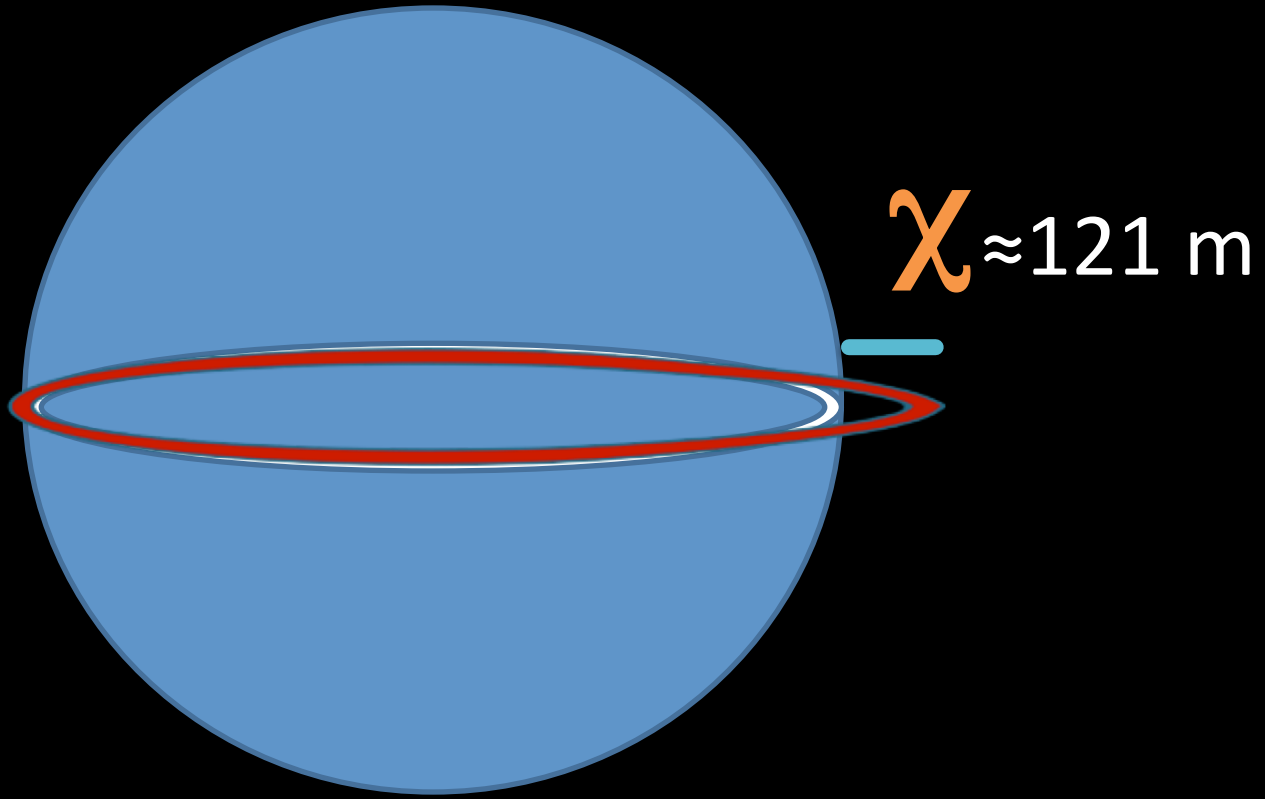



χ

$$2\pi R+1=2\pi(R+x)$$

$$1=2\pi x\Rightarrow x=\frac{1}{2\pi}\approx 0.16$$



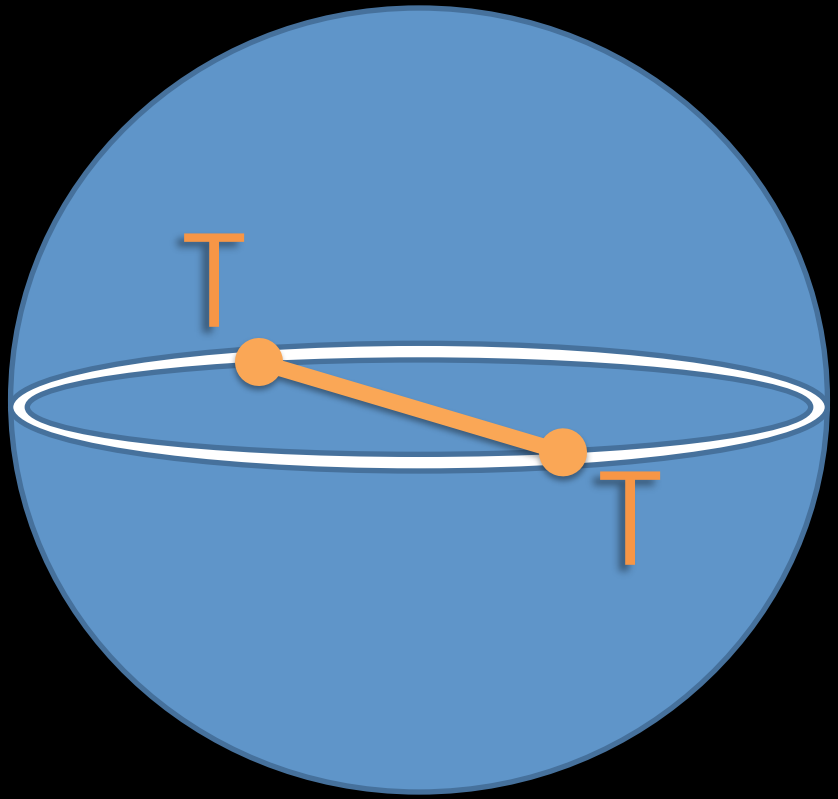

χ≈121 m

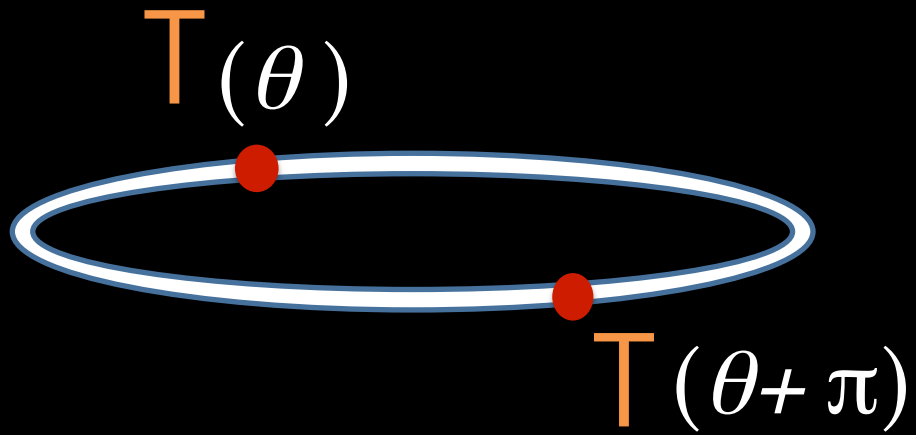
PUZZLE 3

Unreasonable Power of Simple Mathematics

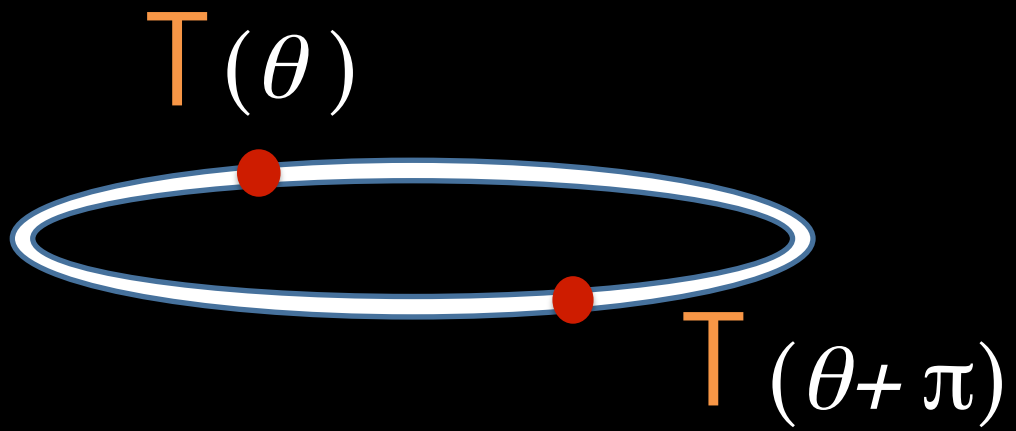
MATH AND PHYSICS CONNECTION:

Power of Continuity

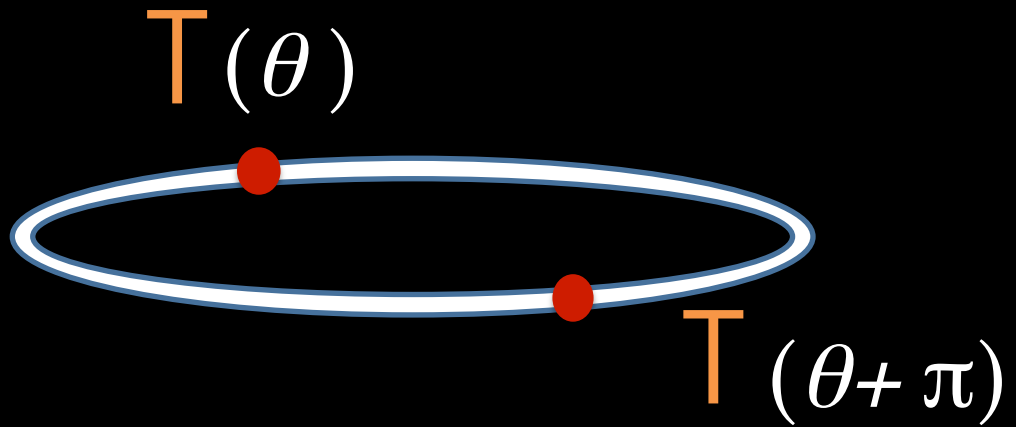

T
T


T (θ)
T (θ+ π)

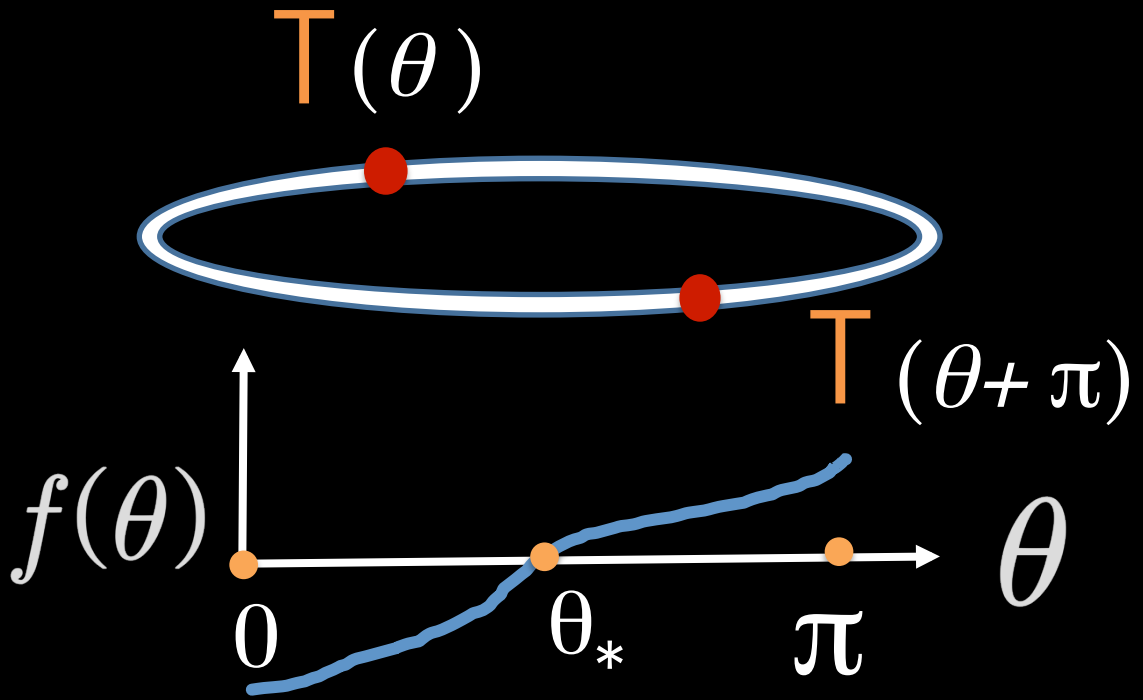
$$f(\theta) \equiv T(\theta) - T(\theta + \pi)$$

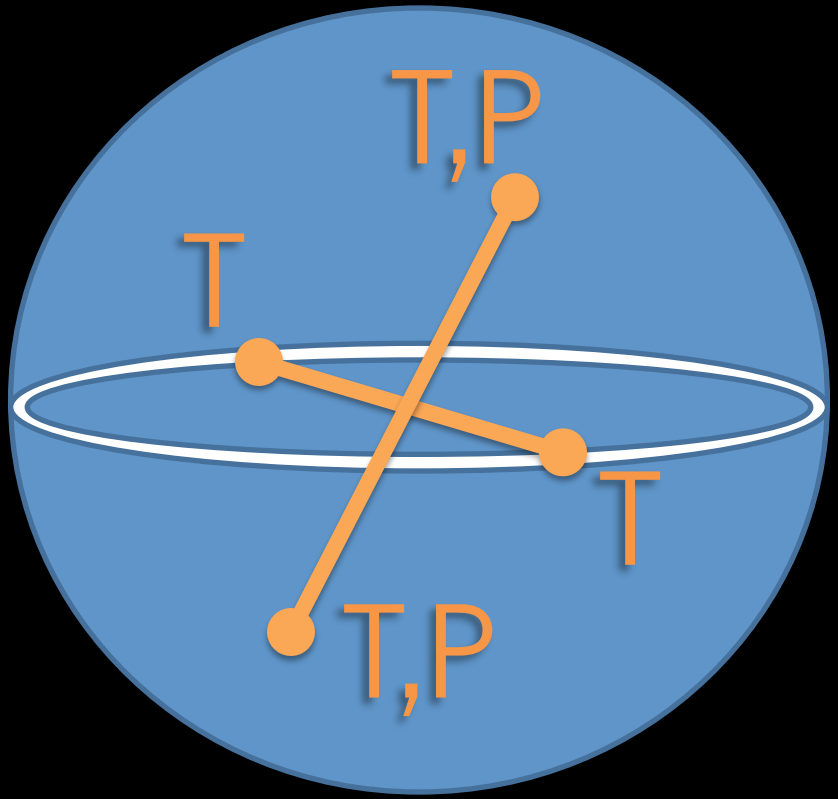


$$f(\theta) \equiv T(\theta) - T(\theta + \pi)$$
$$f(\theta) \equiv -f(\theta + \pi)$$



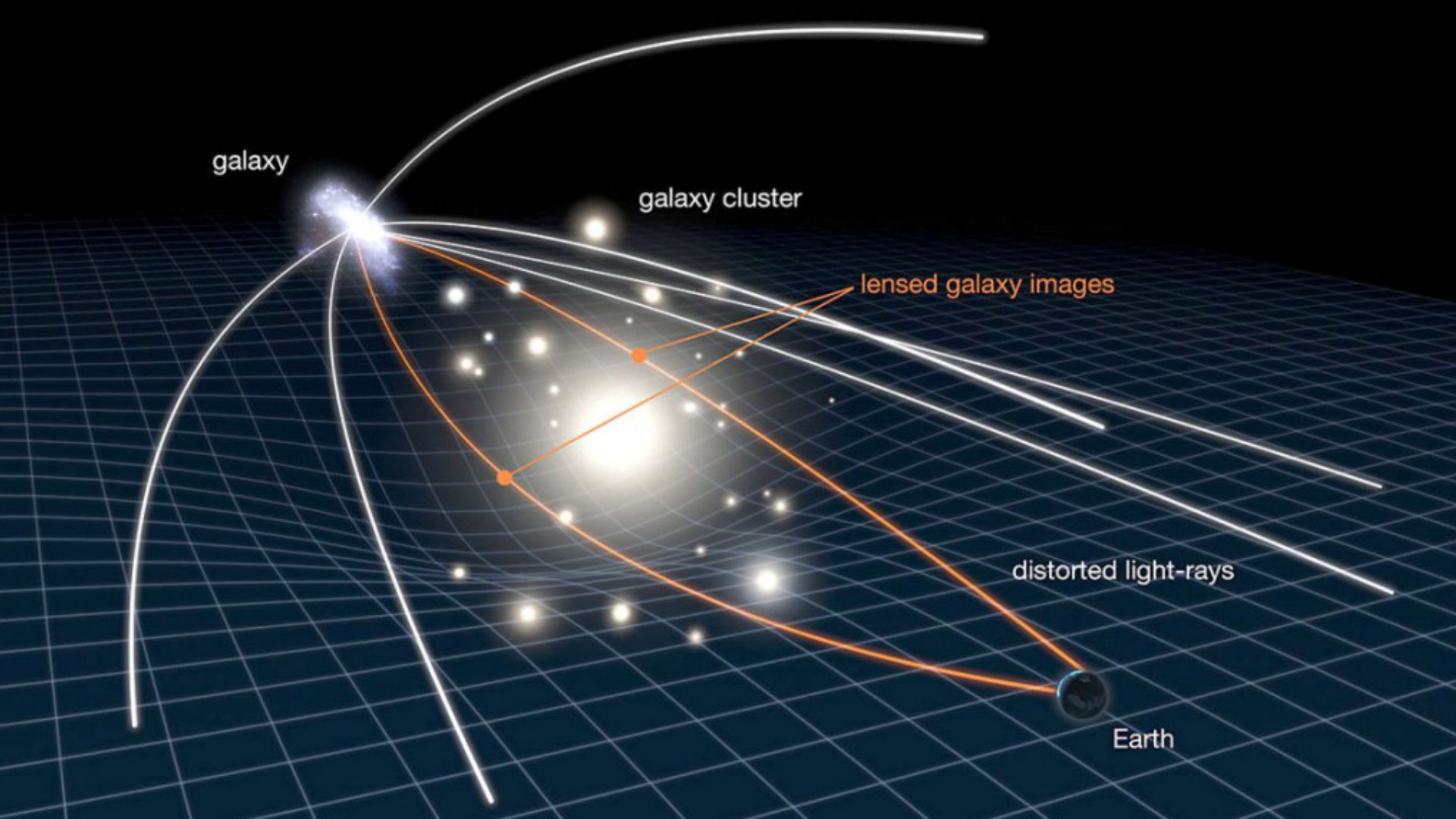
$$f(\theta) \equiv T(\theta) - T(\theta + \pi)$$
$$f(\theta) \equiv -f(\theta + \pi)$$

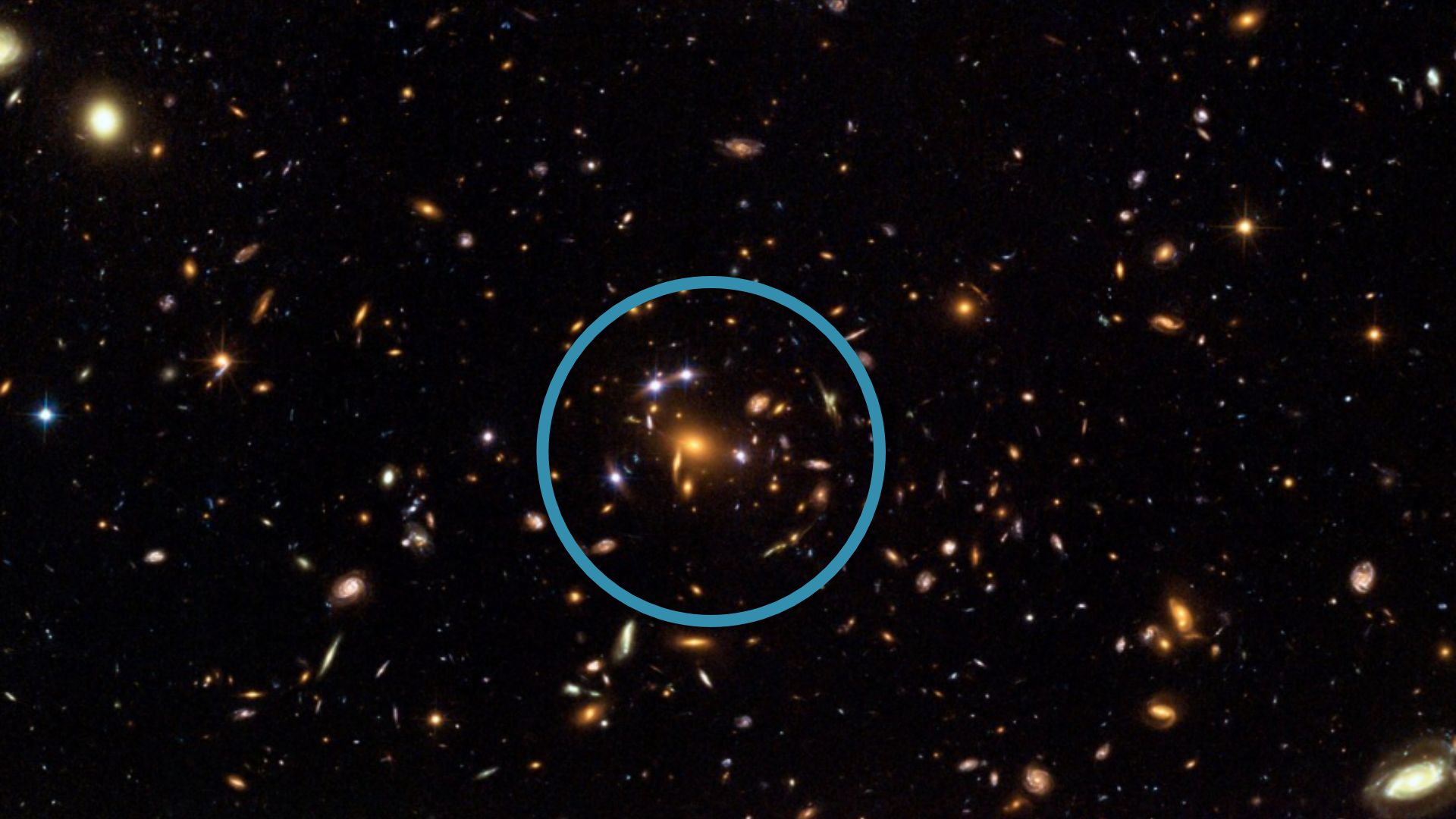



T,P
T
T
T,P

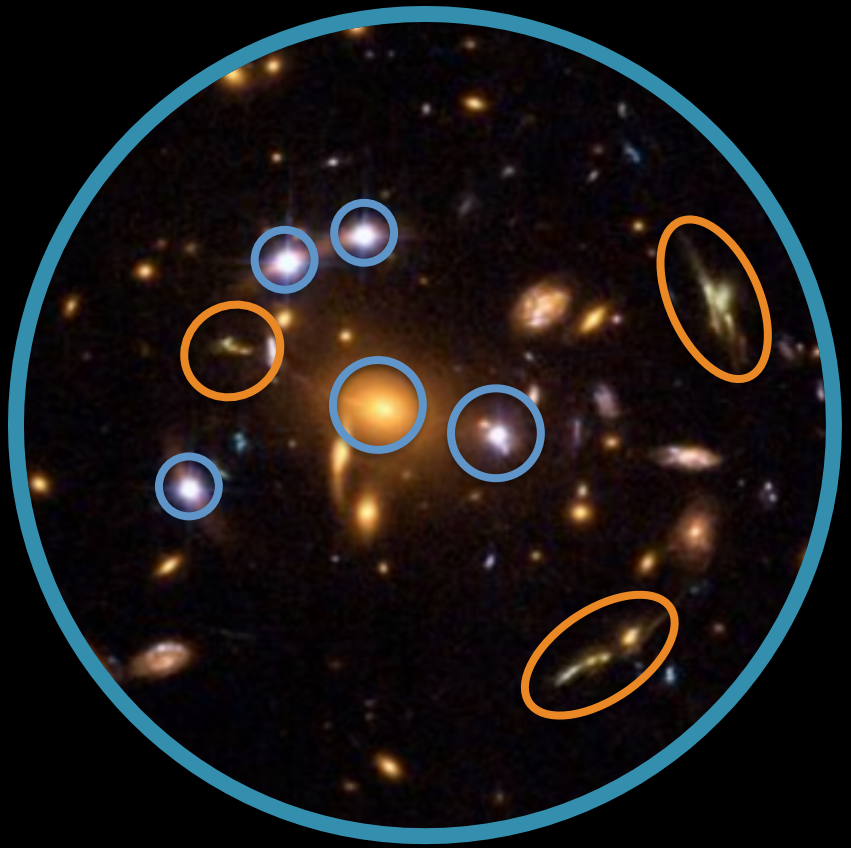
GRAVITATIONAL LENSING

Another example of power of continuity


galaxy
galaxy cluster
lensed galaxy images
distorted light-rays
Earth

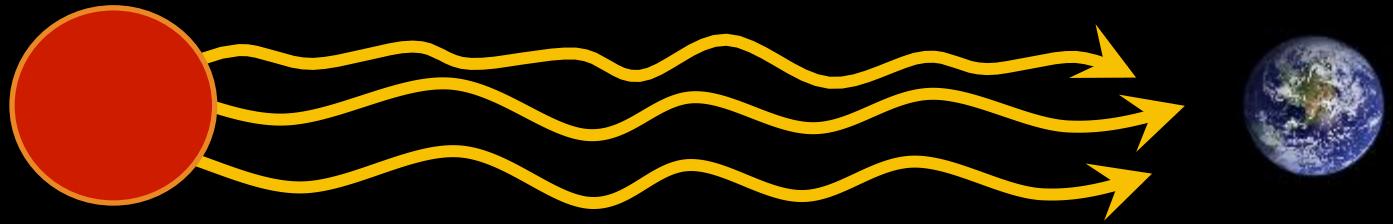


Blue circles: same quasar
Orange circles: same galaxy

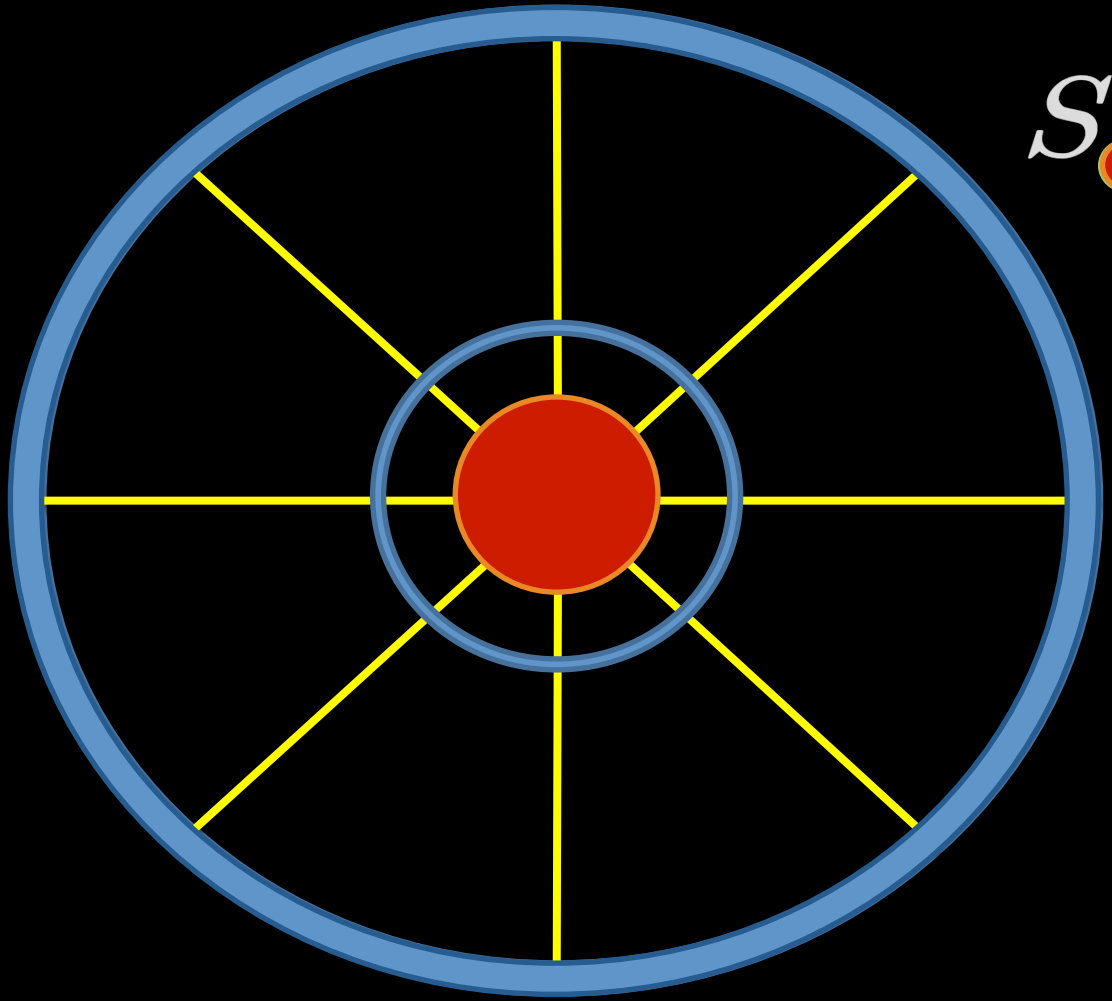


FACT:

The number of gravitational images (if no image is blocked) is always odd. Just less than half of them are inverted images.



When no other matter:



$S^2 \longrightarrow S^2$

DEGREE OF A MAP

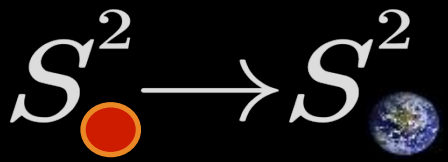
Net number of primages of a given point counted with

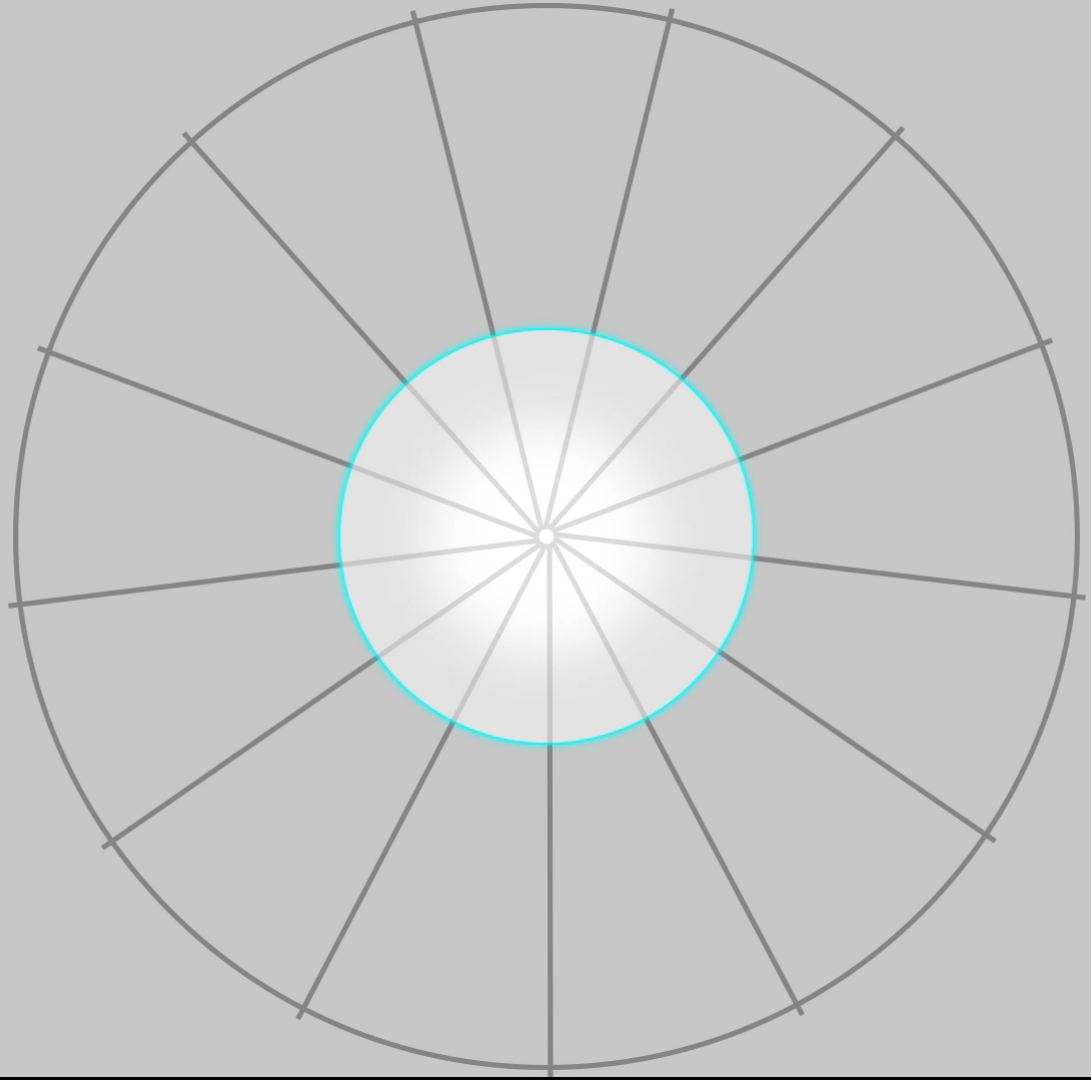
“+” sign if the map is not inverted and “-” sign if it is

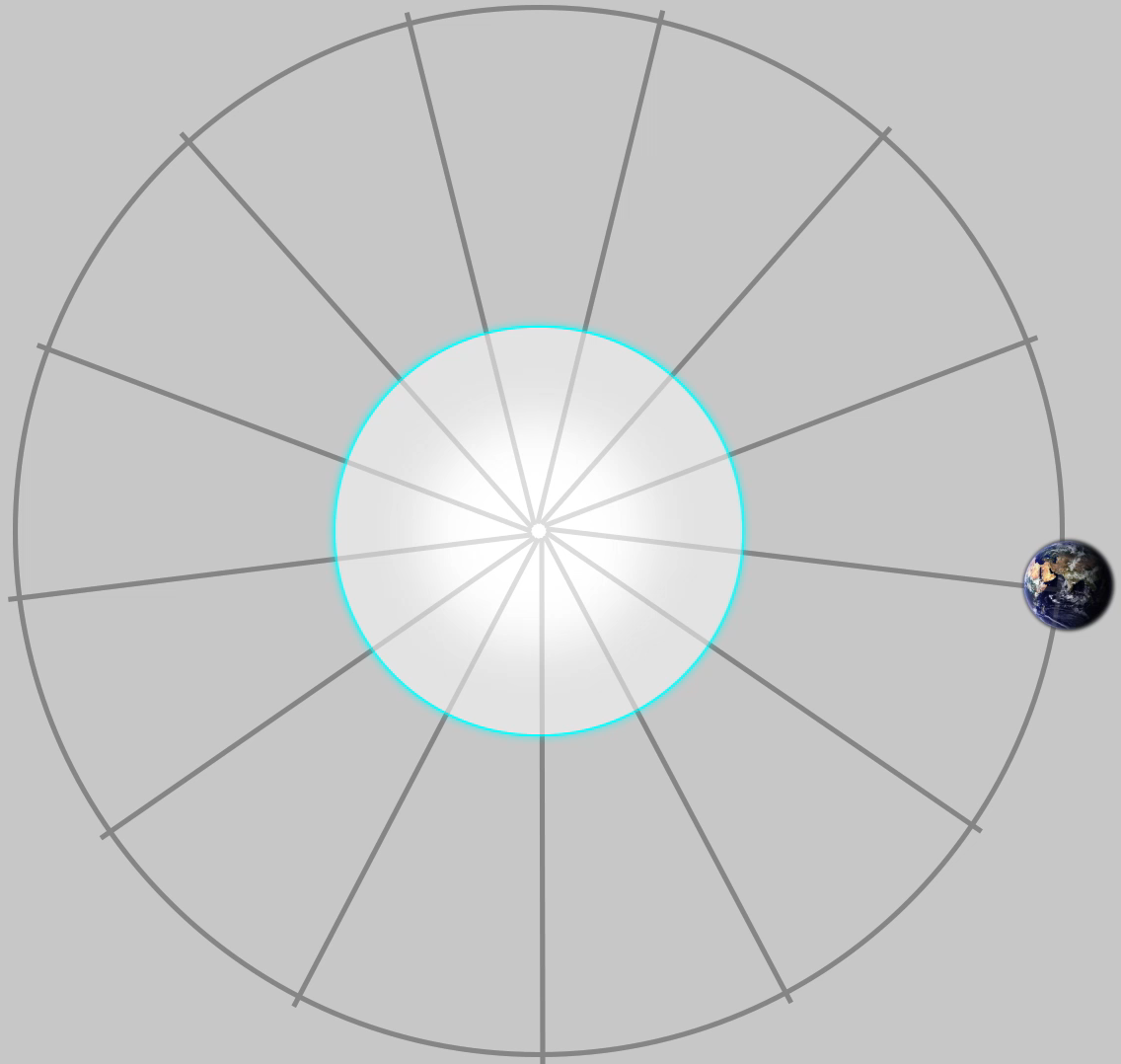
DEGREE OF A MAP

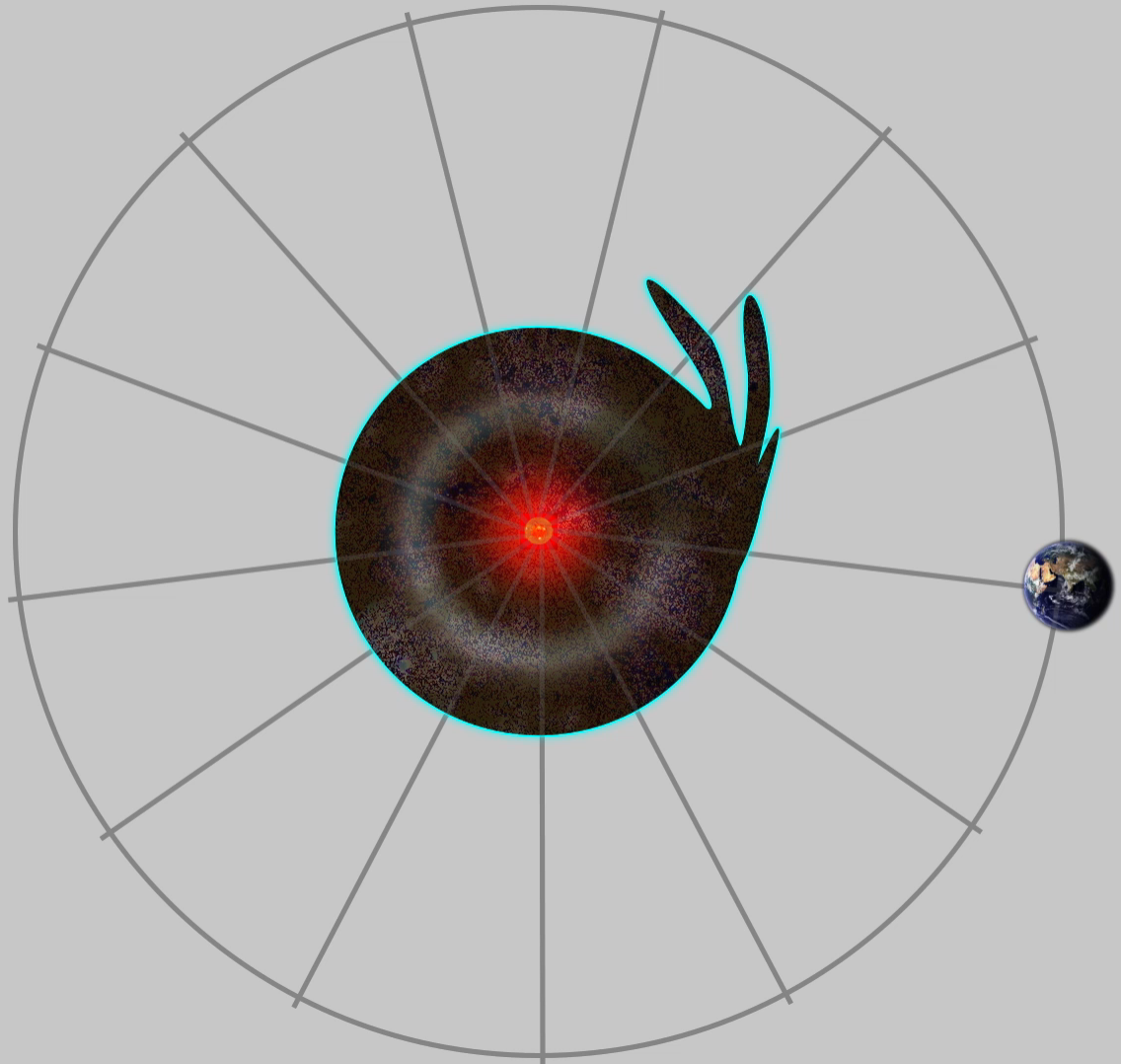
Degree of the map

when there is no matter is 1









PUZZLE 4

Ants Colliding


ANT 1
ANT 2
ANT 3
ANT 4

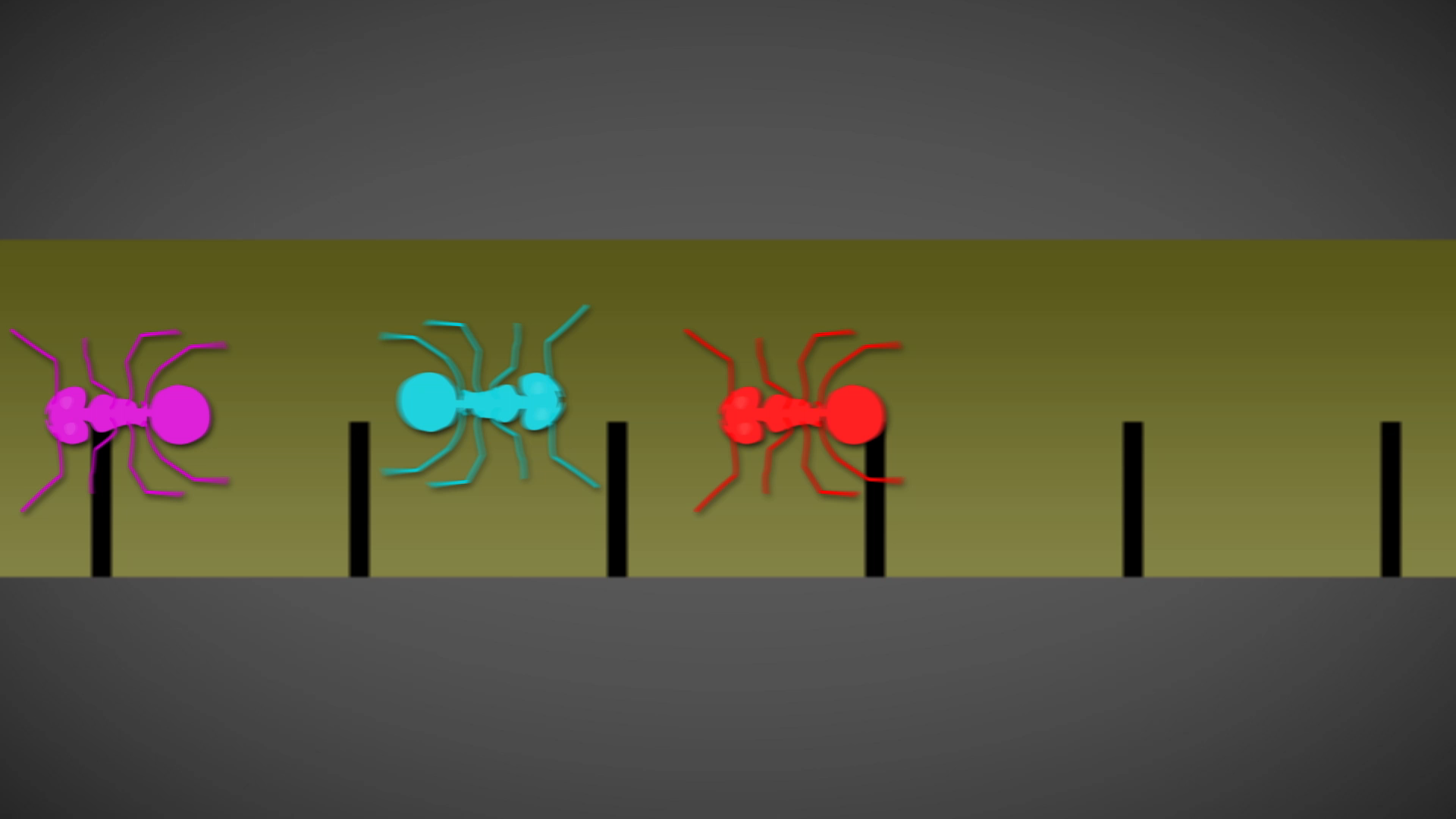

4
3
2
1

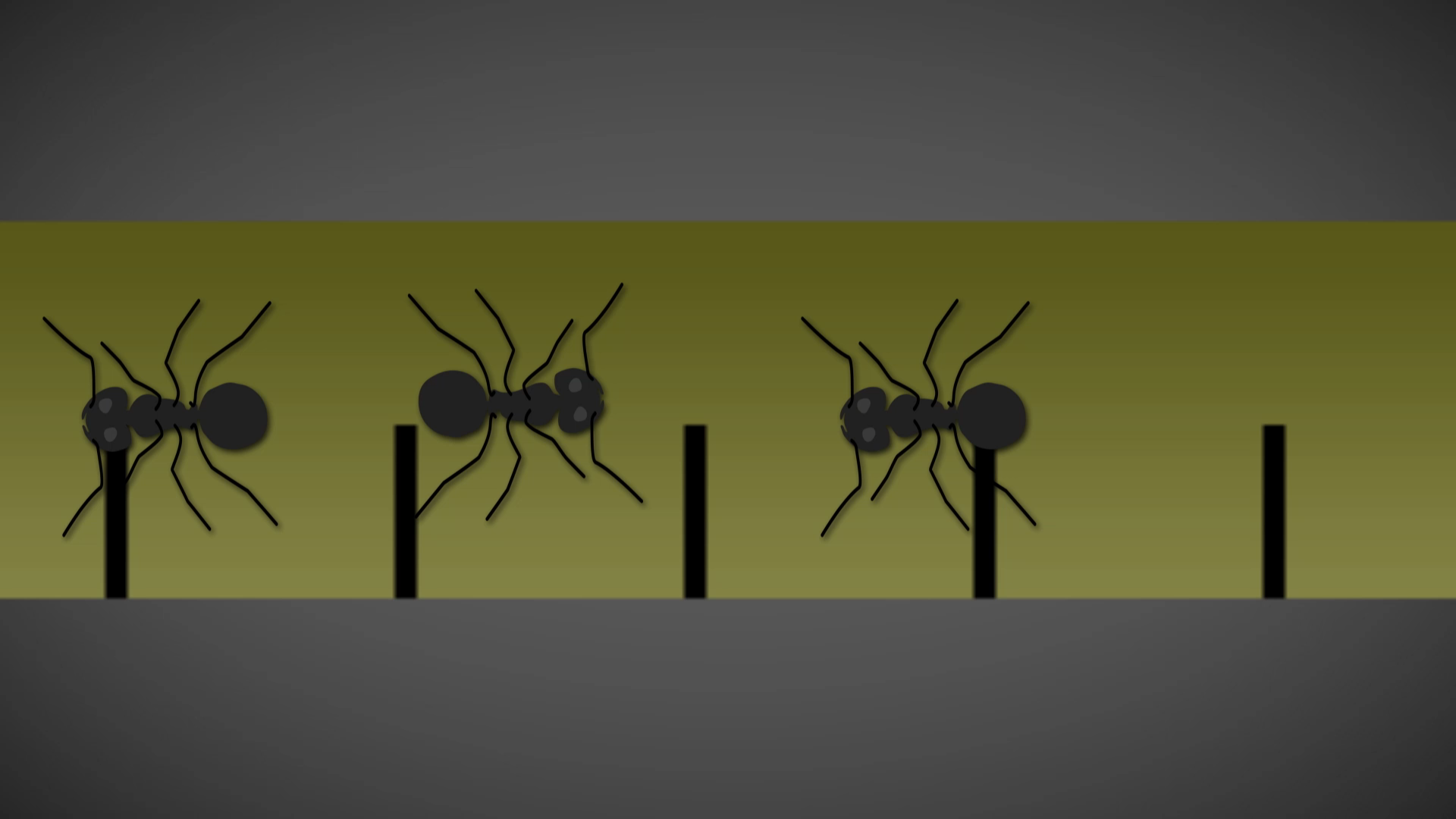
PUZZLE 4

Math and Physics Connection:
Power of Mathematical Abstraction

PUZZLE 5

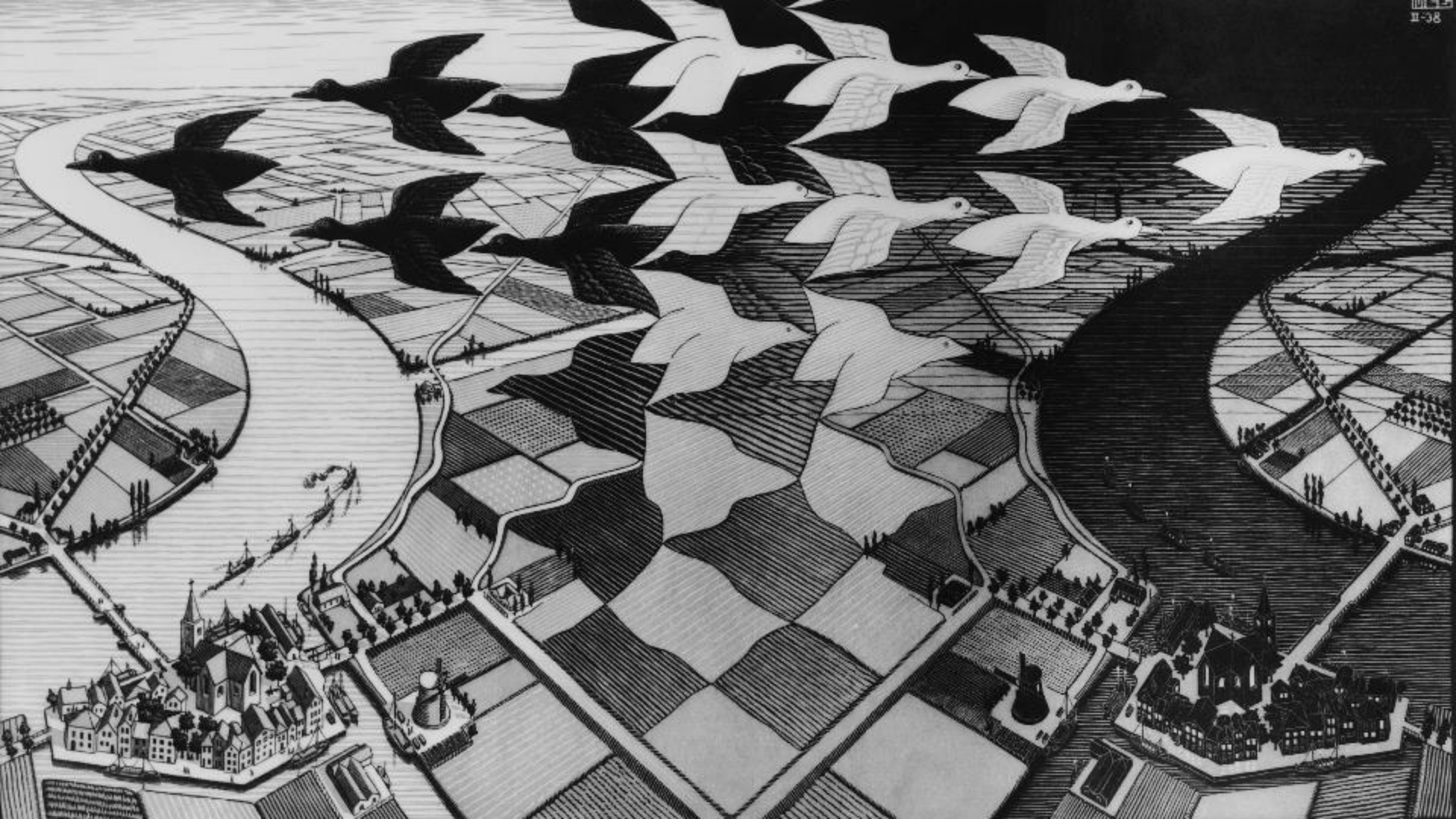
Ants on a Meter Stick





PUZZLE 5

Math and Physics Connection: Duality



PUZZLE 5

Math and Physics Connection: Duality

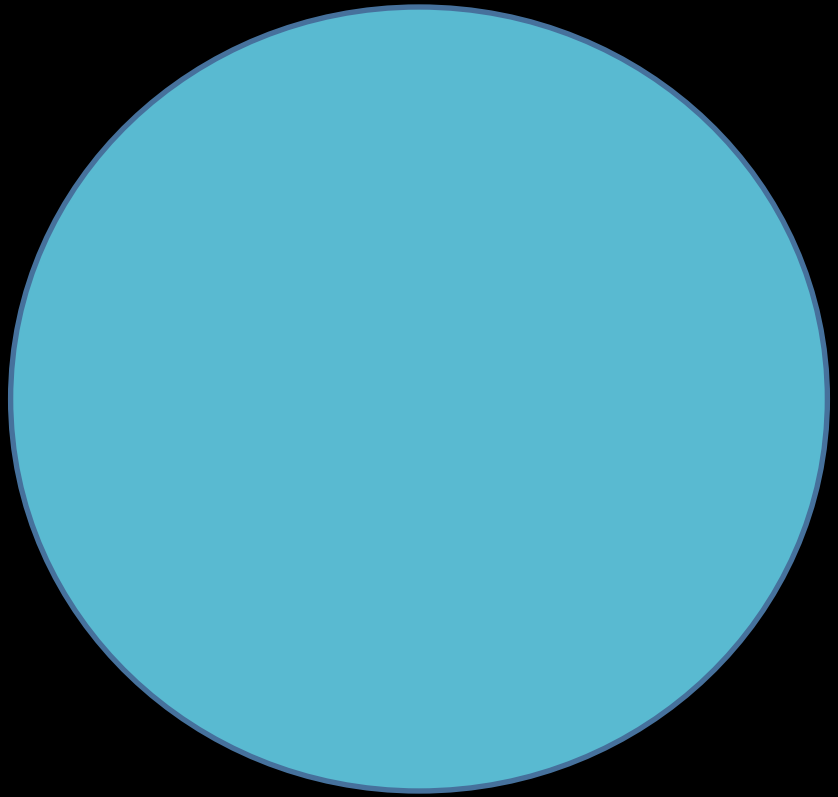
Two seemingly different systems can nevertheless be identical.
This typically involves a change of perspective.

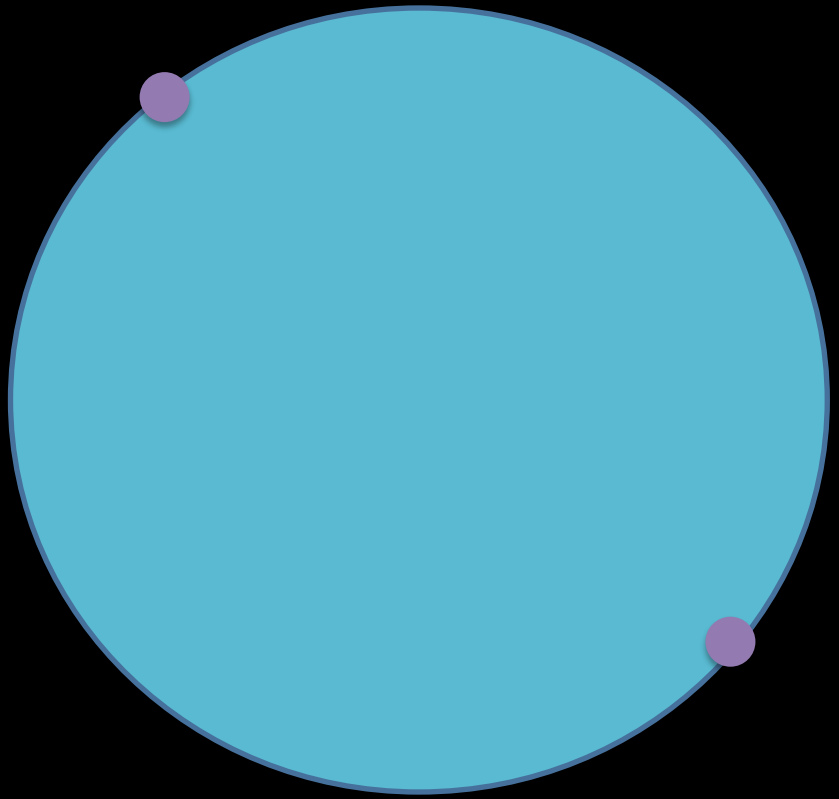
PUZZLE 6

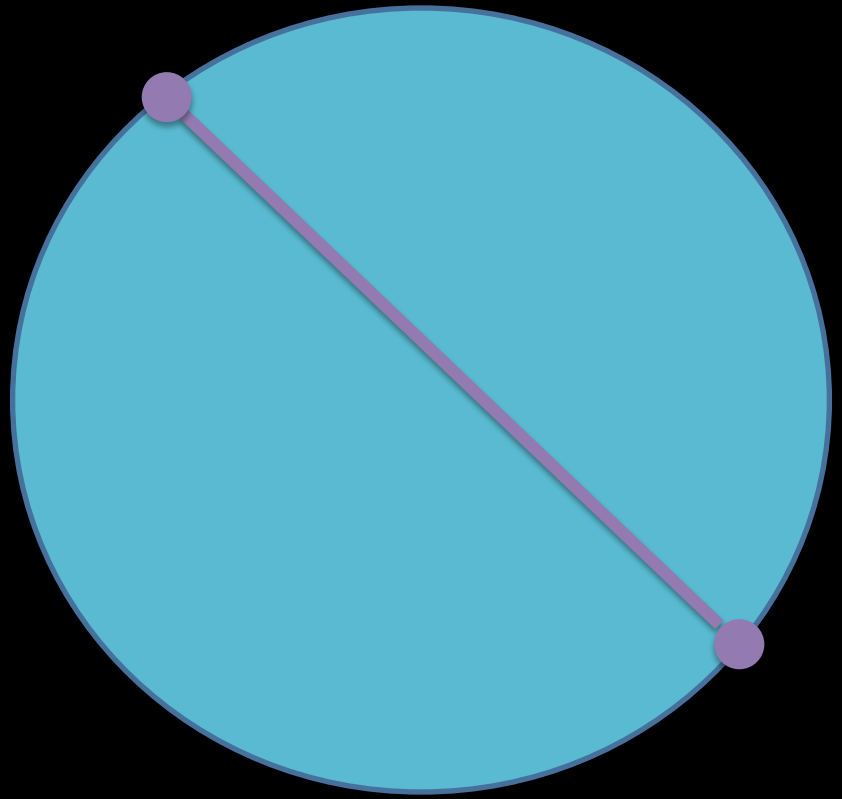
Points and Regions

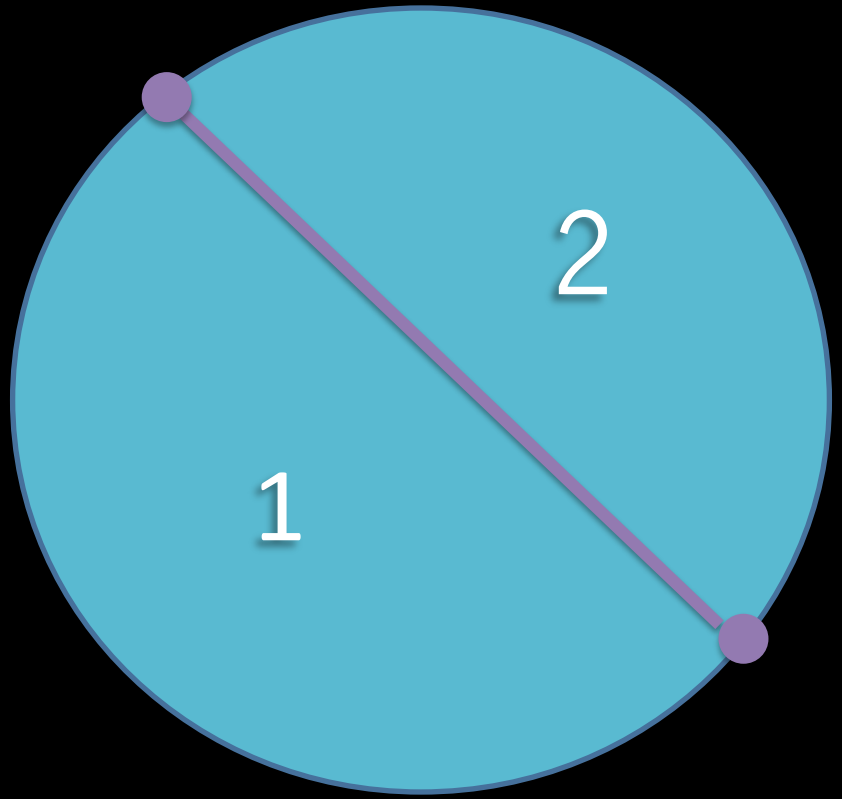
REFLECTIONS ON SCIENTIFIC METHODOLOGY

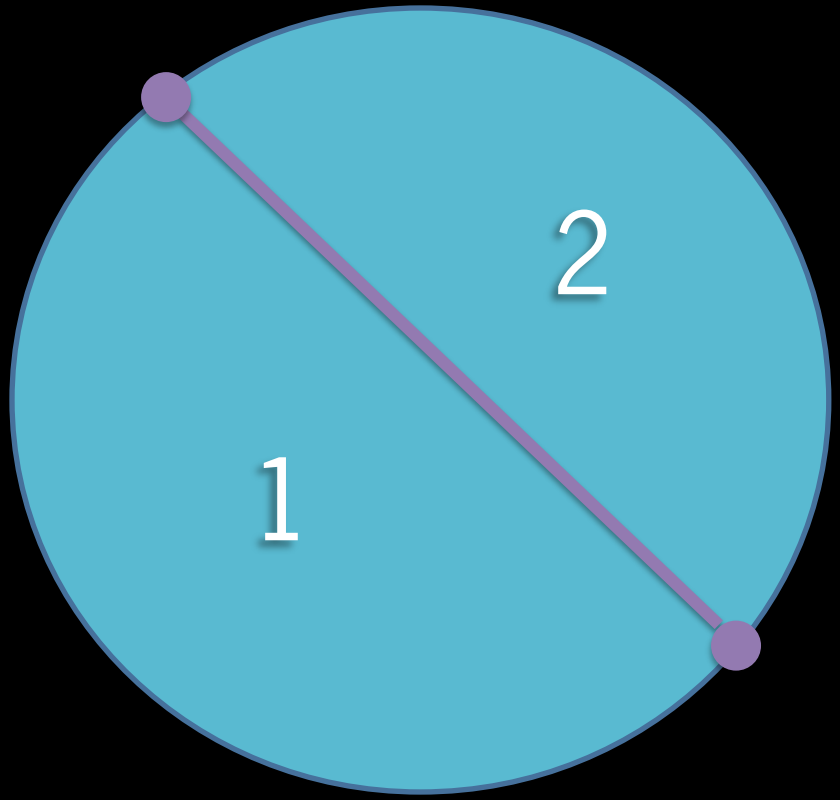
1. Examples/experiments
2. Formulate a general principle based on examples
3. Come up with arguments why/how it works…



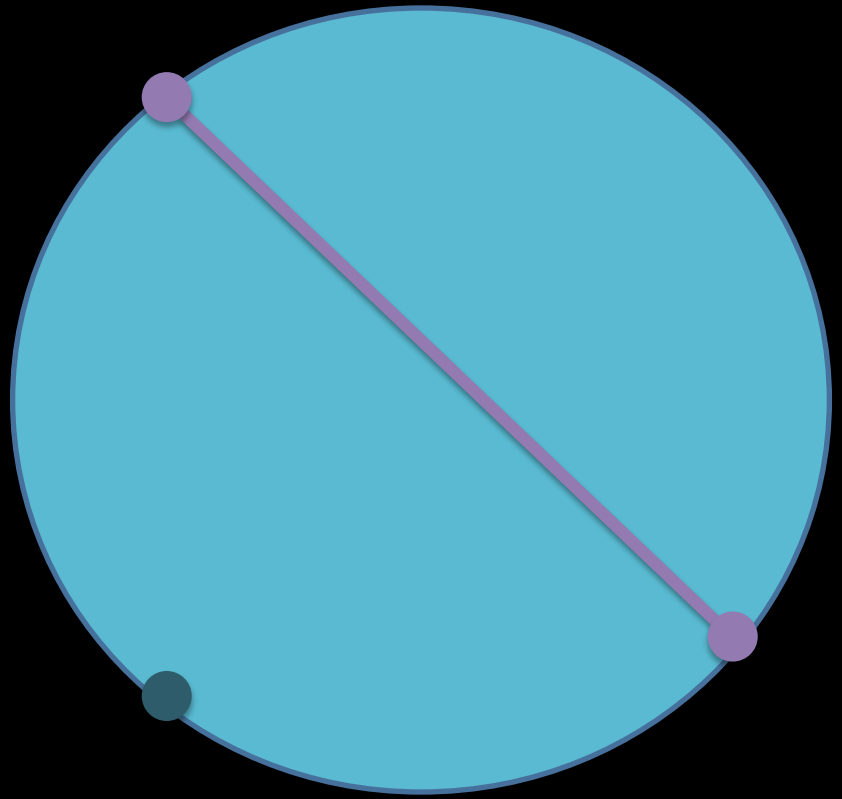


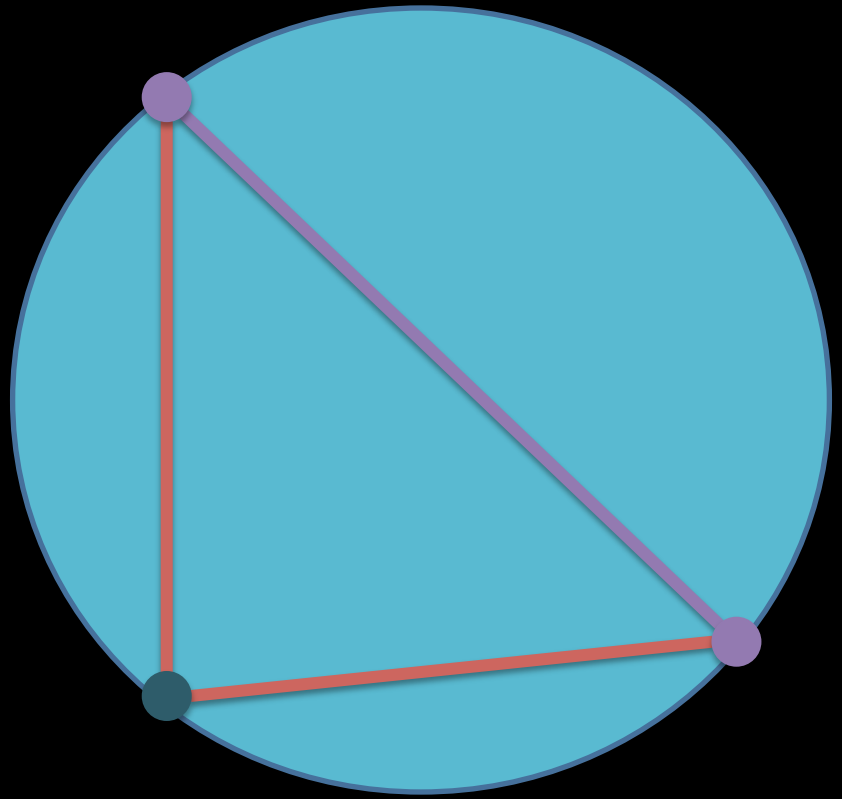


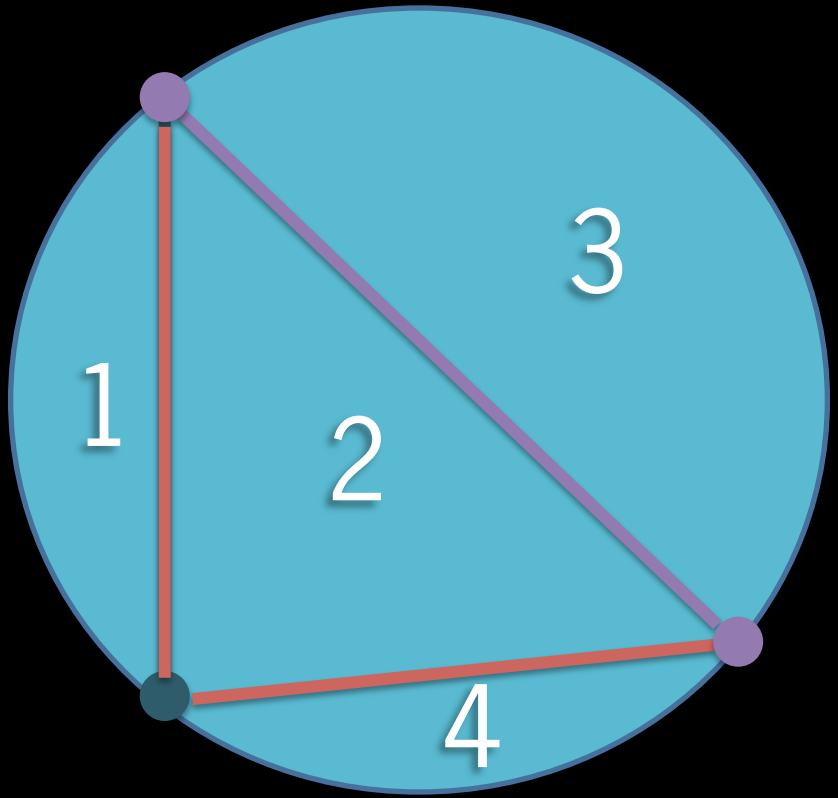

2
1

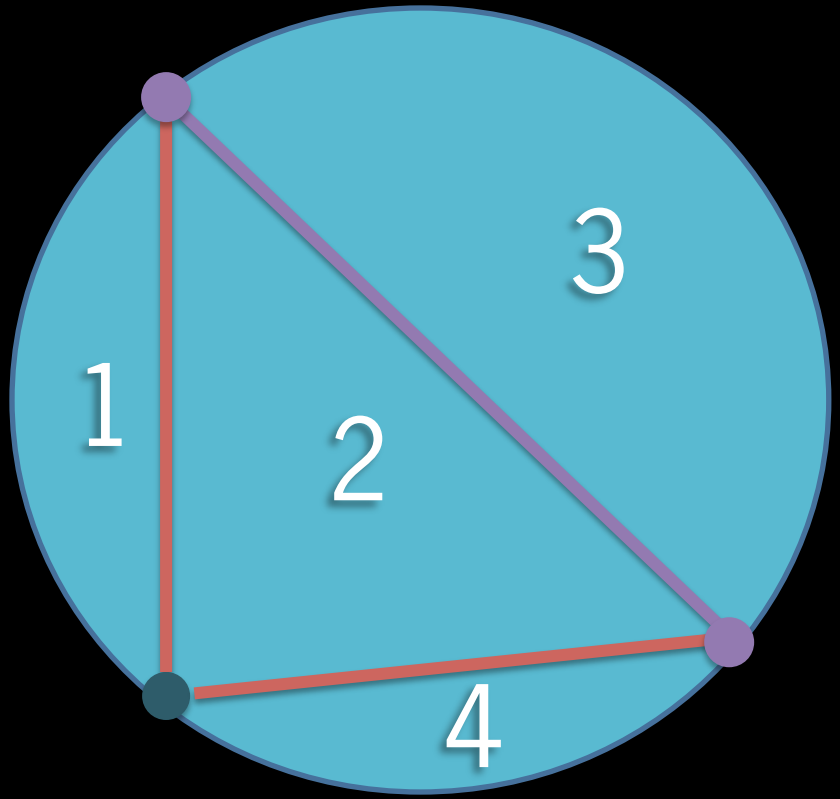


# Points	# Regions
2	2

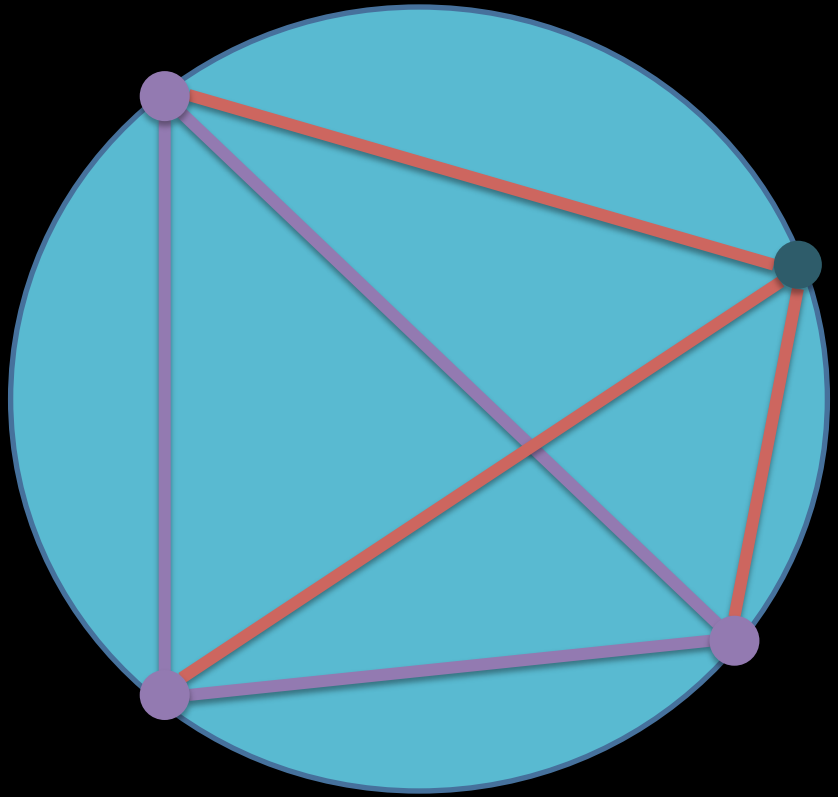


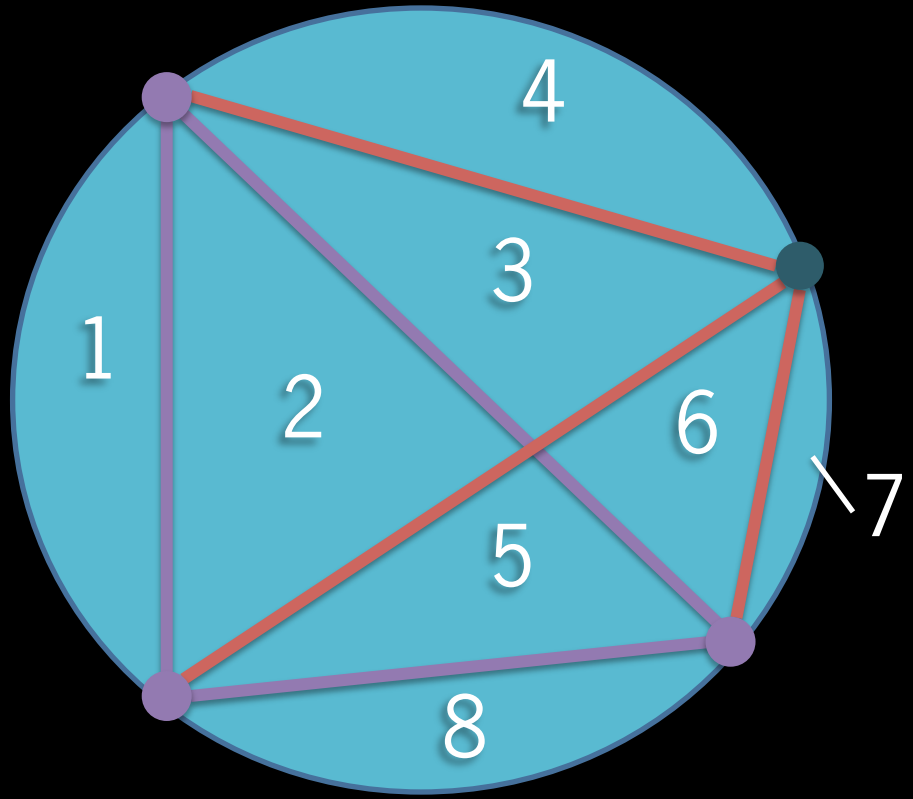


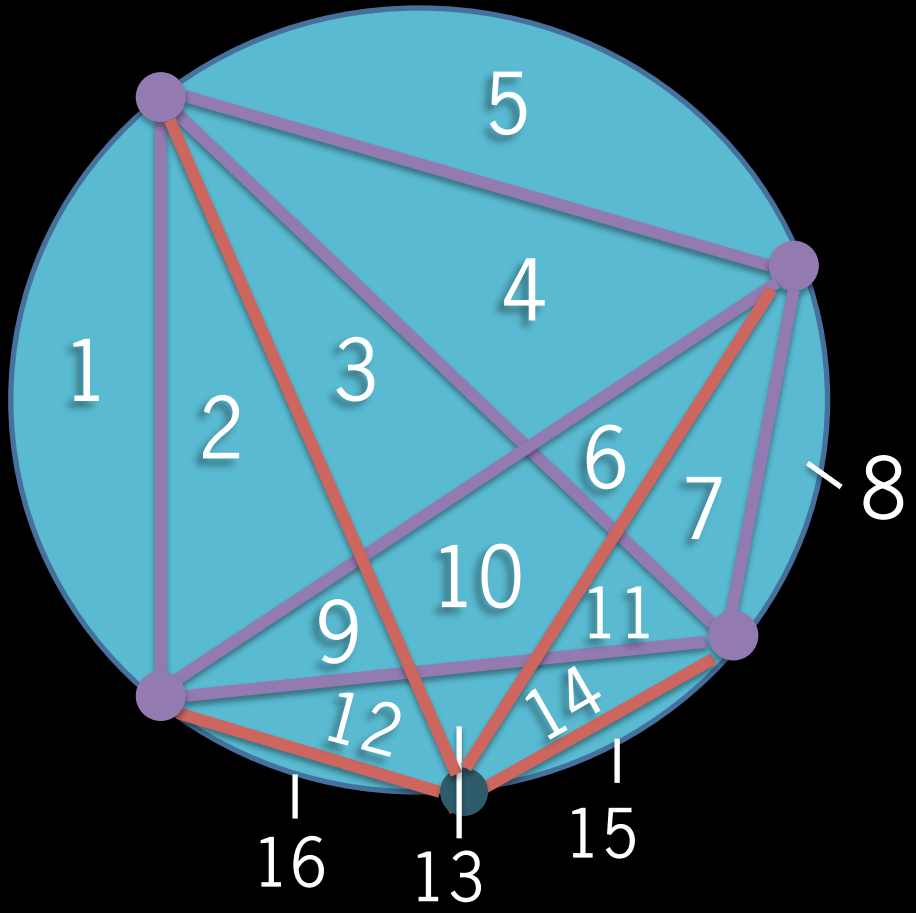

1
2
3
4



# Points	# Regions
2	2
3	4




4
3
1
2
6
7
5
8


5
4
1
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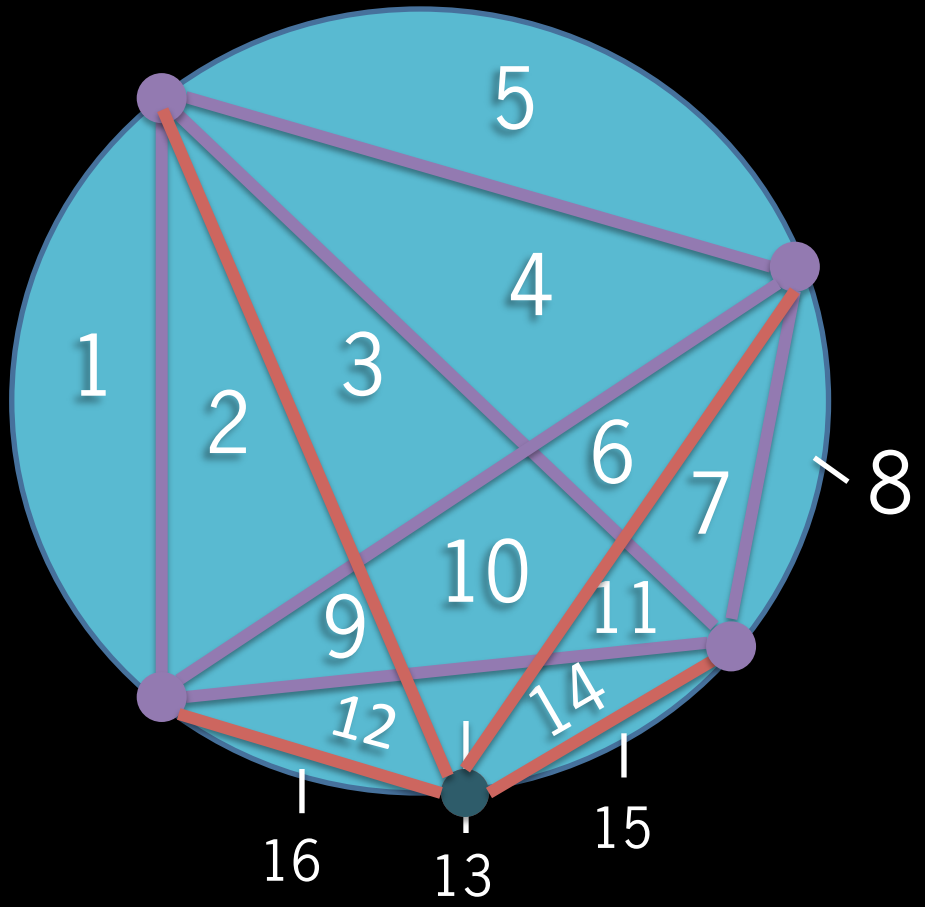
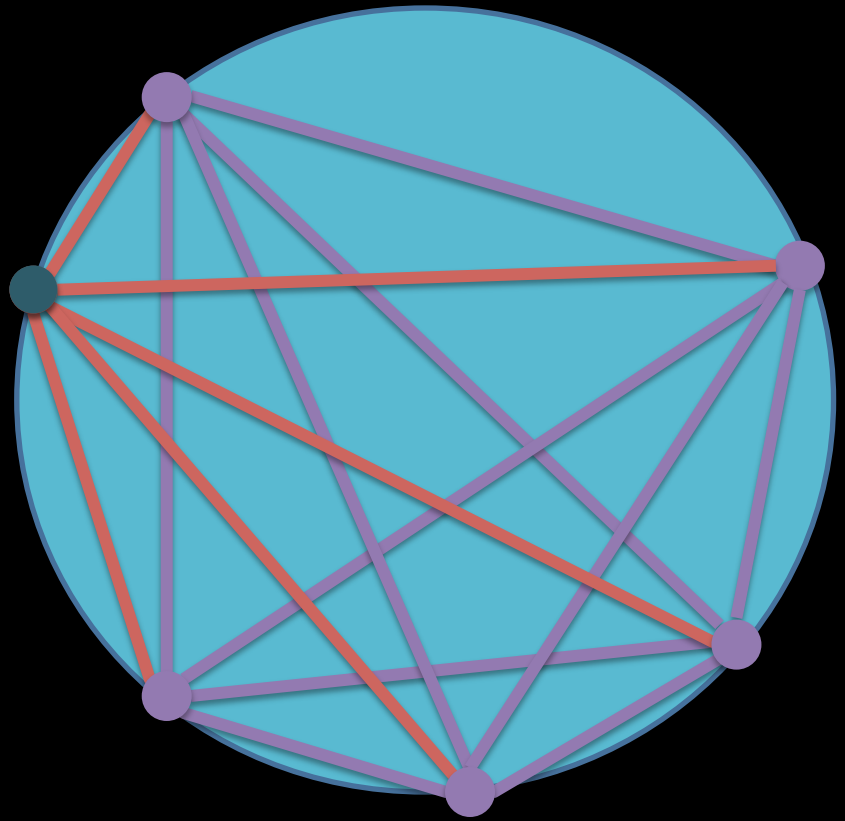
14
16
13
15

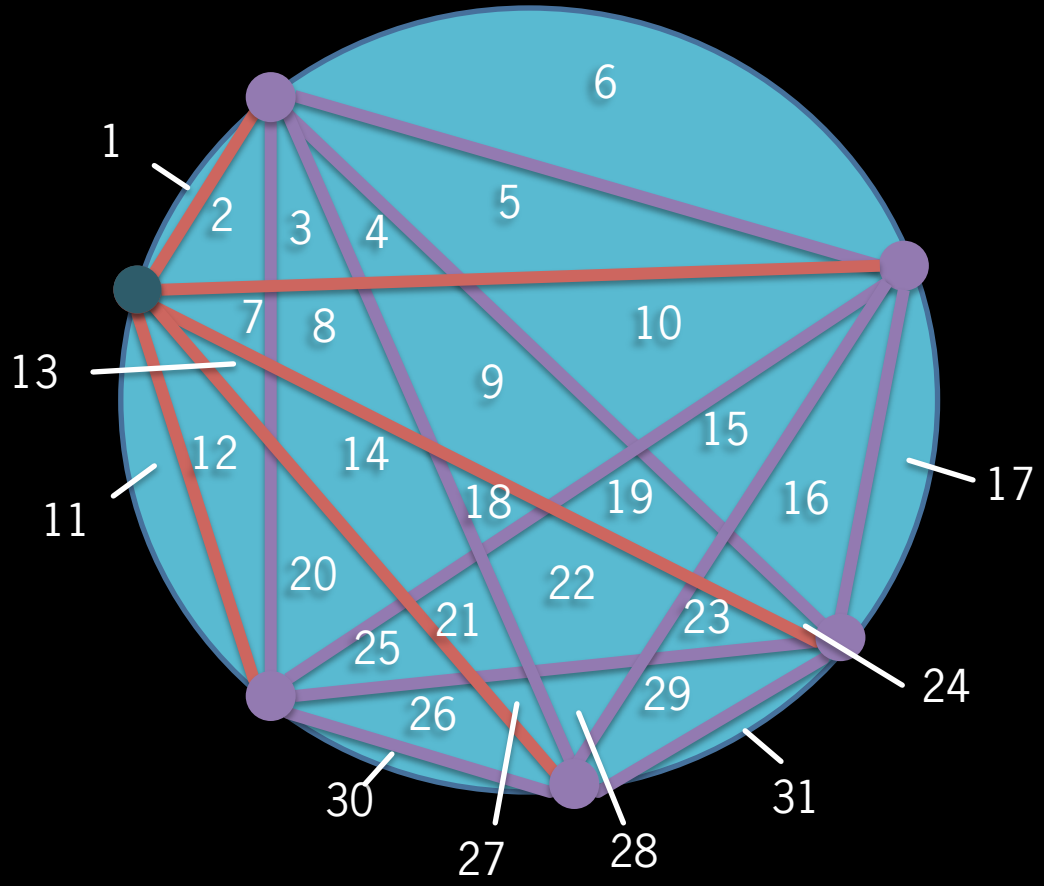


# Points	# Regions
2	2
3	4
4	8
5	16

WHAT IS THE EXPLANATION?

# Points	# Regions
2	2
3	4
4	8
5	16




1
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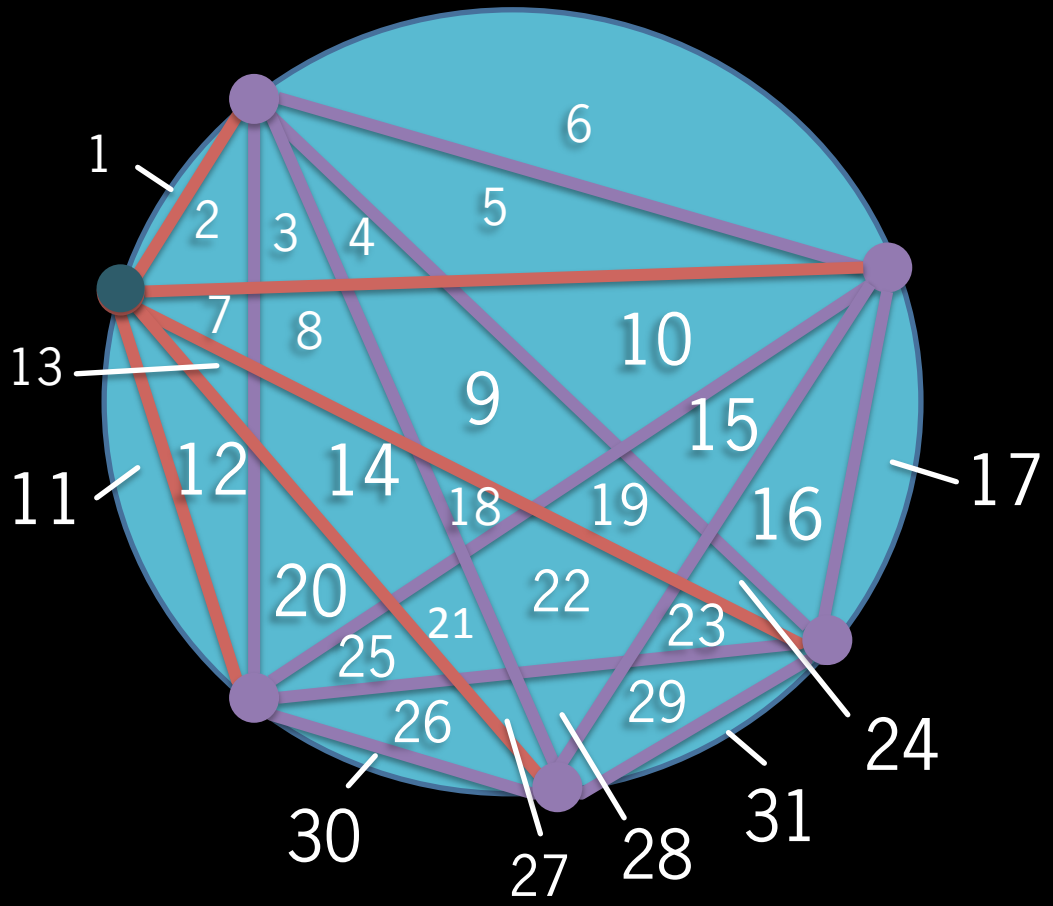

# Points	# Regions
2	2
3	4
4	8
5	16
6	31

$$R = 1 + \binom{N}{2} + \binom{N}{4}$$

# Points	# Regions
2	2
3	4
4	8
5	16
6	31

MATHEMATICAL PUZZLES

Encapsulate deep physical principles

And they’re fun!

I hope this encourages you not only to have **fun** with solving puzzles, but also to ask what **nugget of truth** we learn from each one.