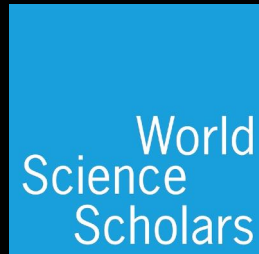


MODULE 3

A BEAUTIFUL UNIVERSE:

Black Holes, String Theory,
and the Laws of Nature as Mathematical Puzzles



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Gravity + quantum mechanical description of particles



?

STORY III:



Emergence of string theory
(Late 1960's continuing till now)

Unifying Einstein's geometric theory of gravity and quantum mechanics had been a nightmare...

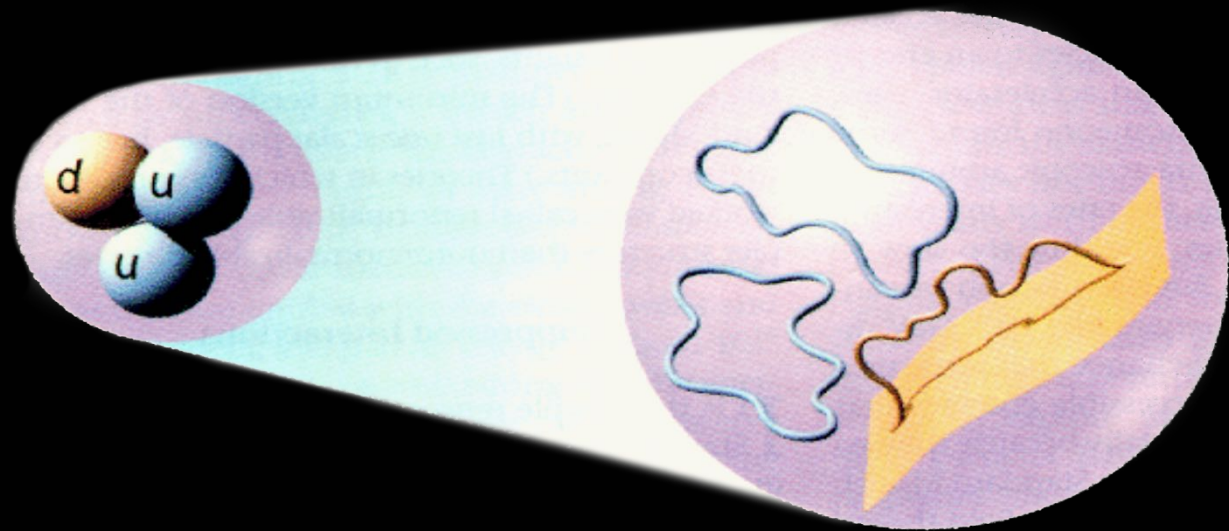
If we treat the particle which carries gravitational force as point-like, quantum computations lead to infinities when we try to compute physical quantities.

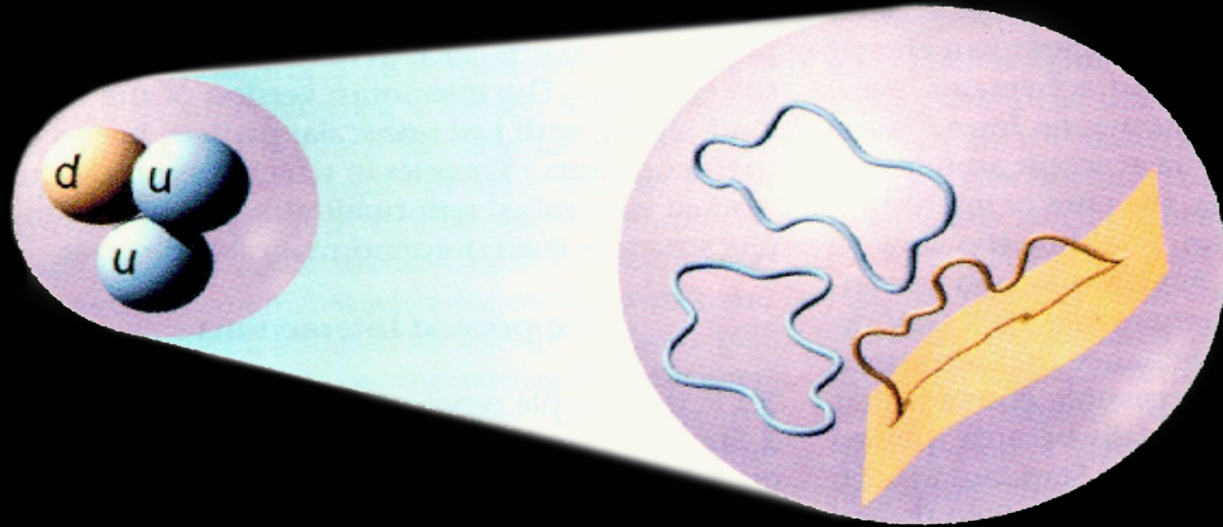
Not acceptable!



RESOLUTION

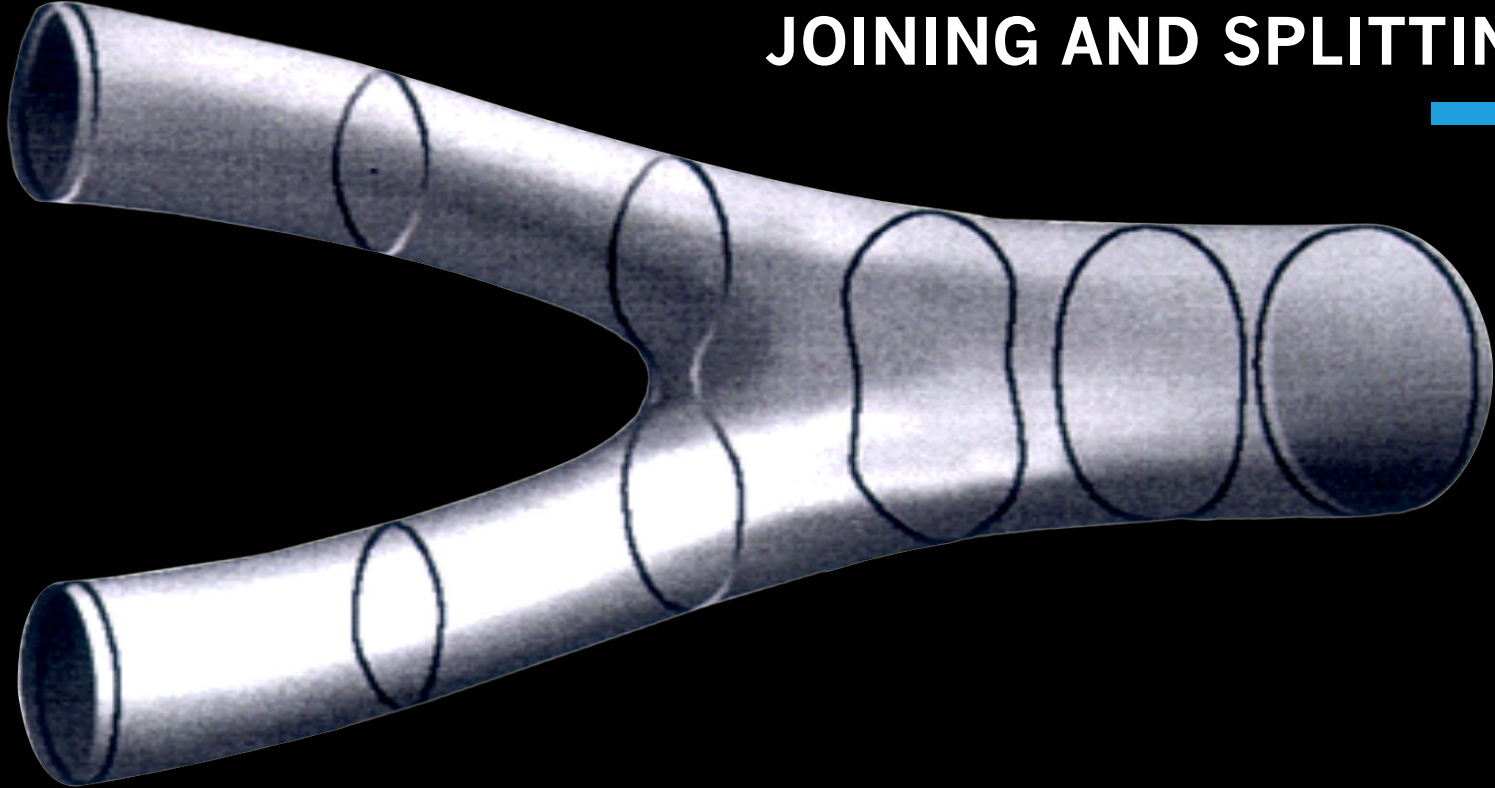


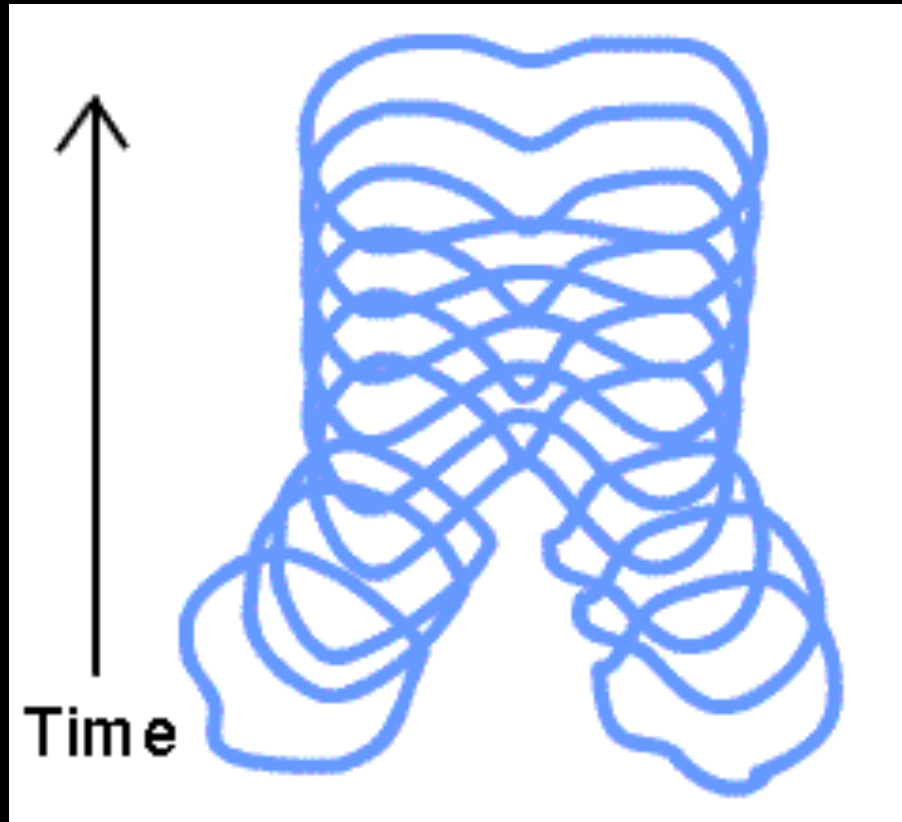




Resolves the inconsistency between quantum theory and gravity

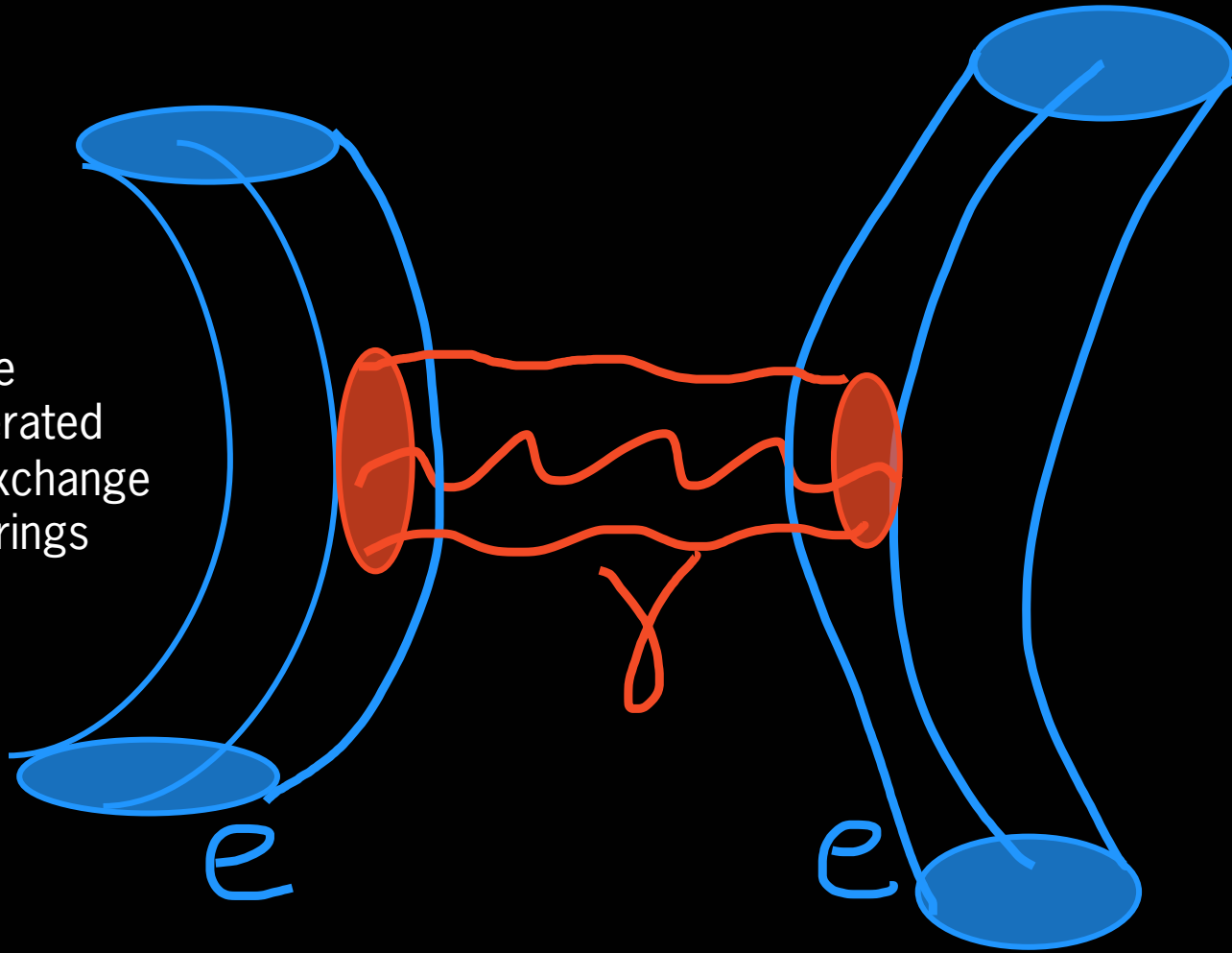
STRINGS INTERACT BY JOINING AND SPLITTING



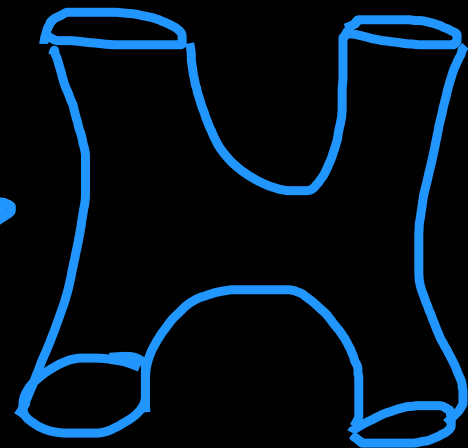
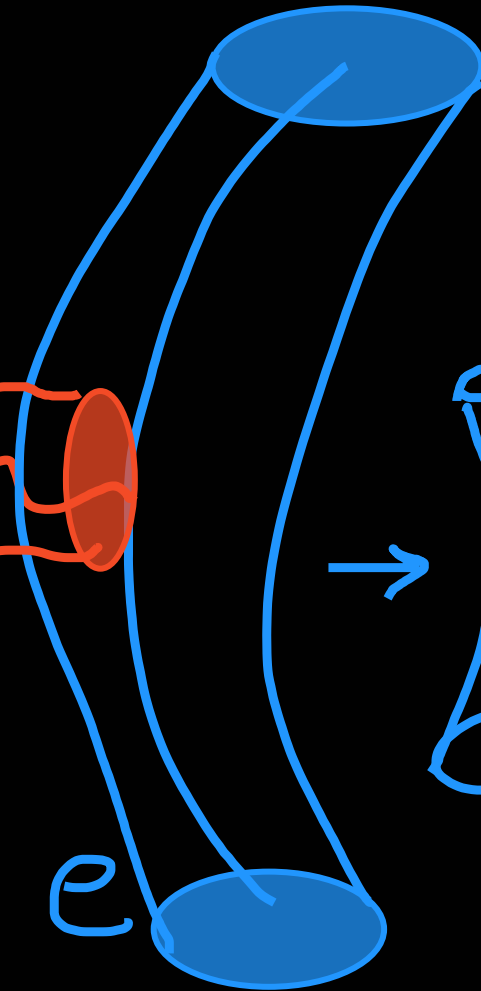
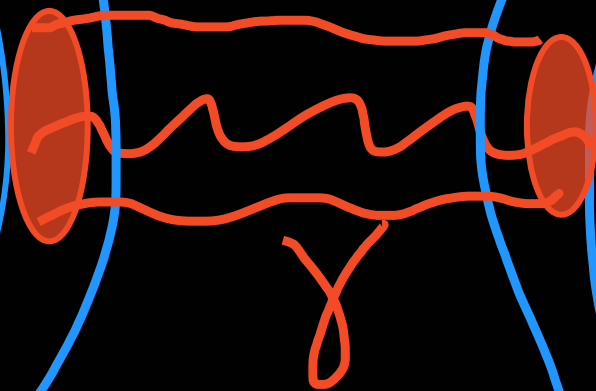
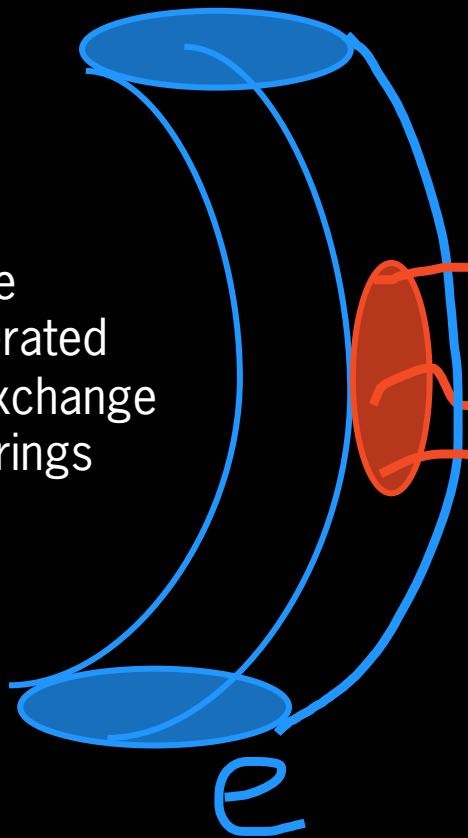


JOINING OF STRINGS

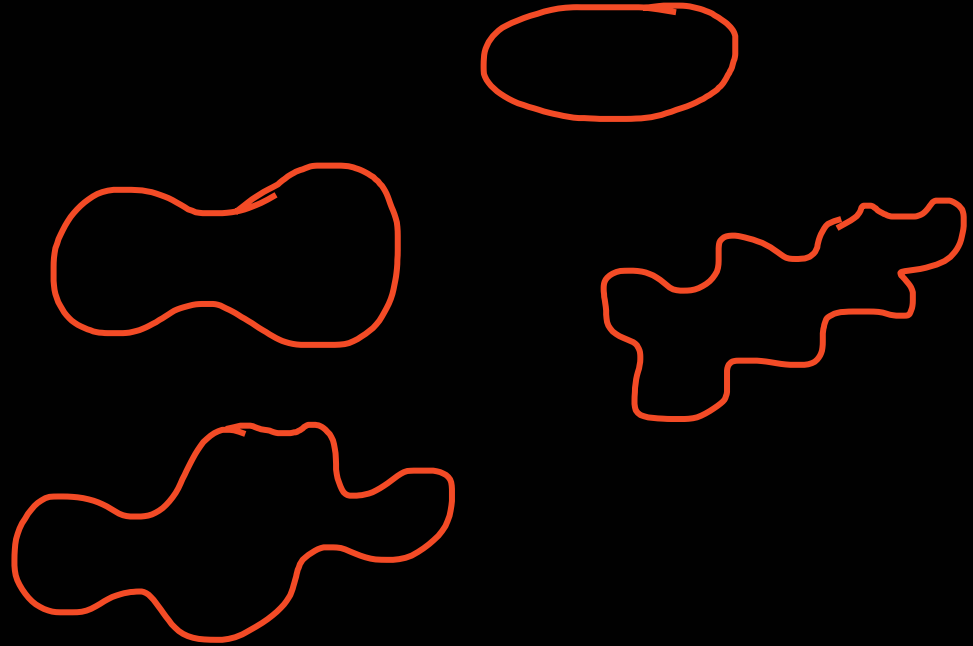
Force
generated
by exchange
of strings

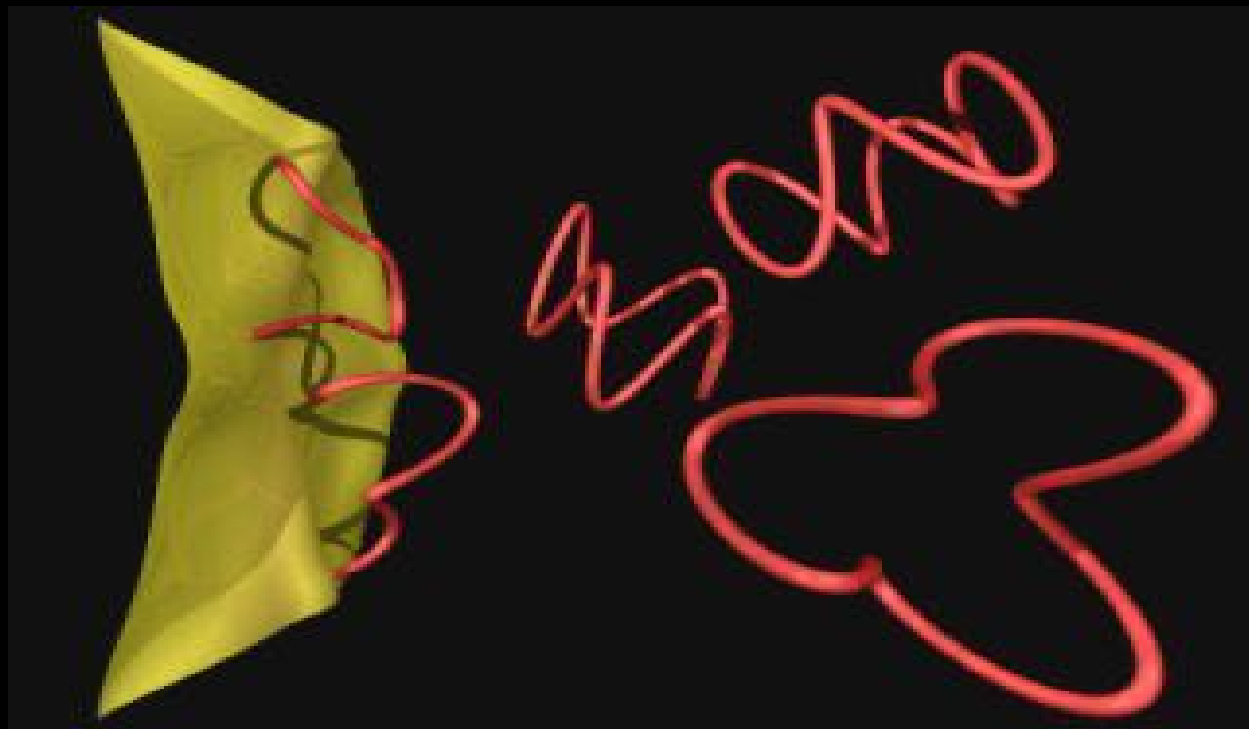


Force
generated
by exchange
of strings



String unifies all
particles into one:





It also has extended objects in addition to strings

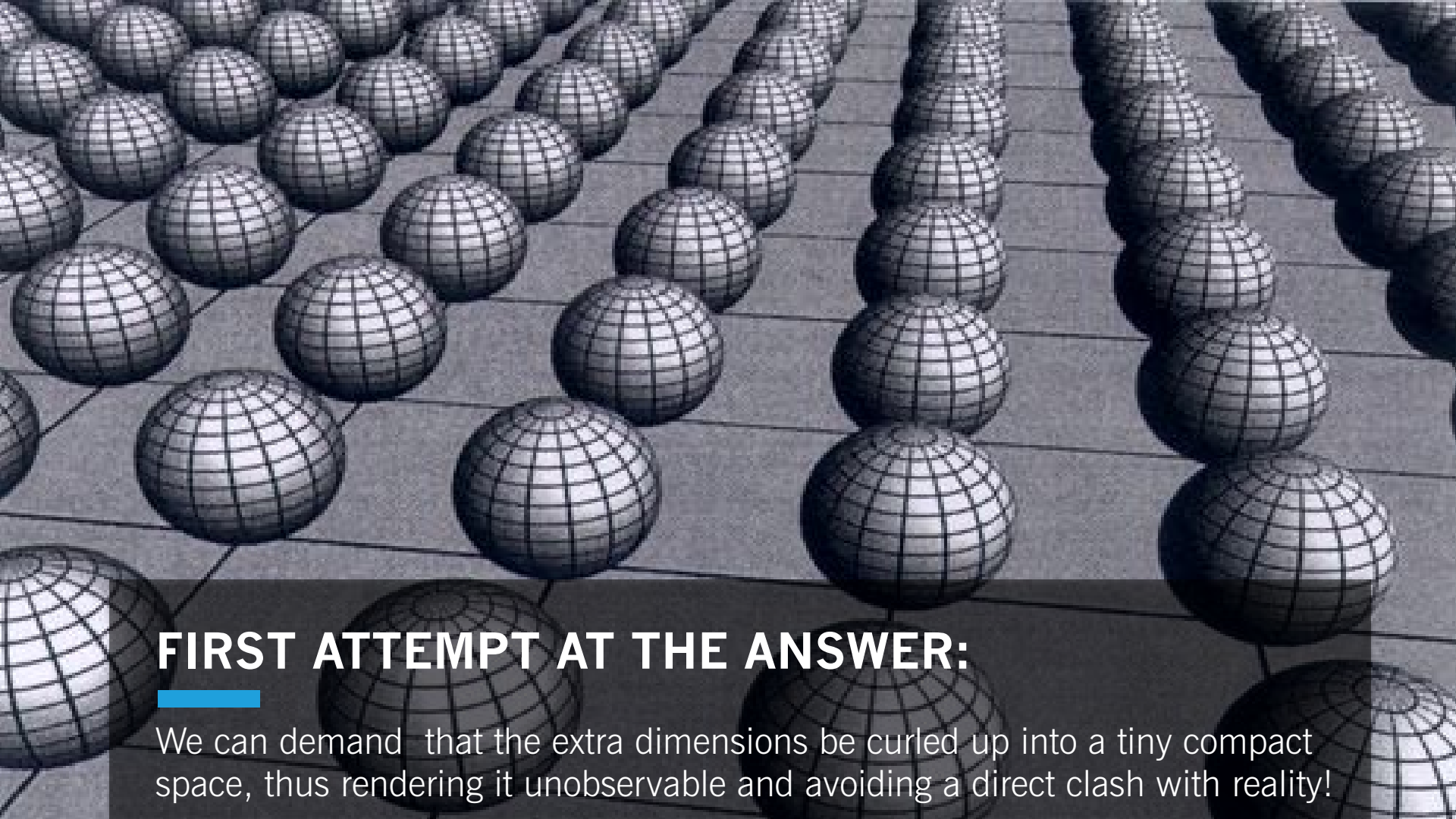
Consistency of string theory:

Dimension of spacetime is more than $3+1$!

It requires $9+1$ dimensions for consistency!

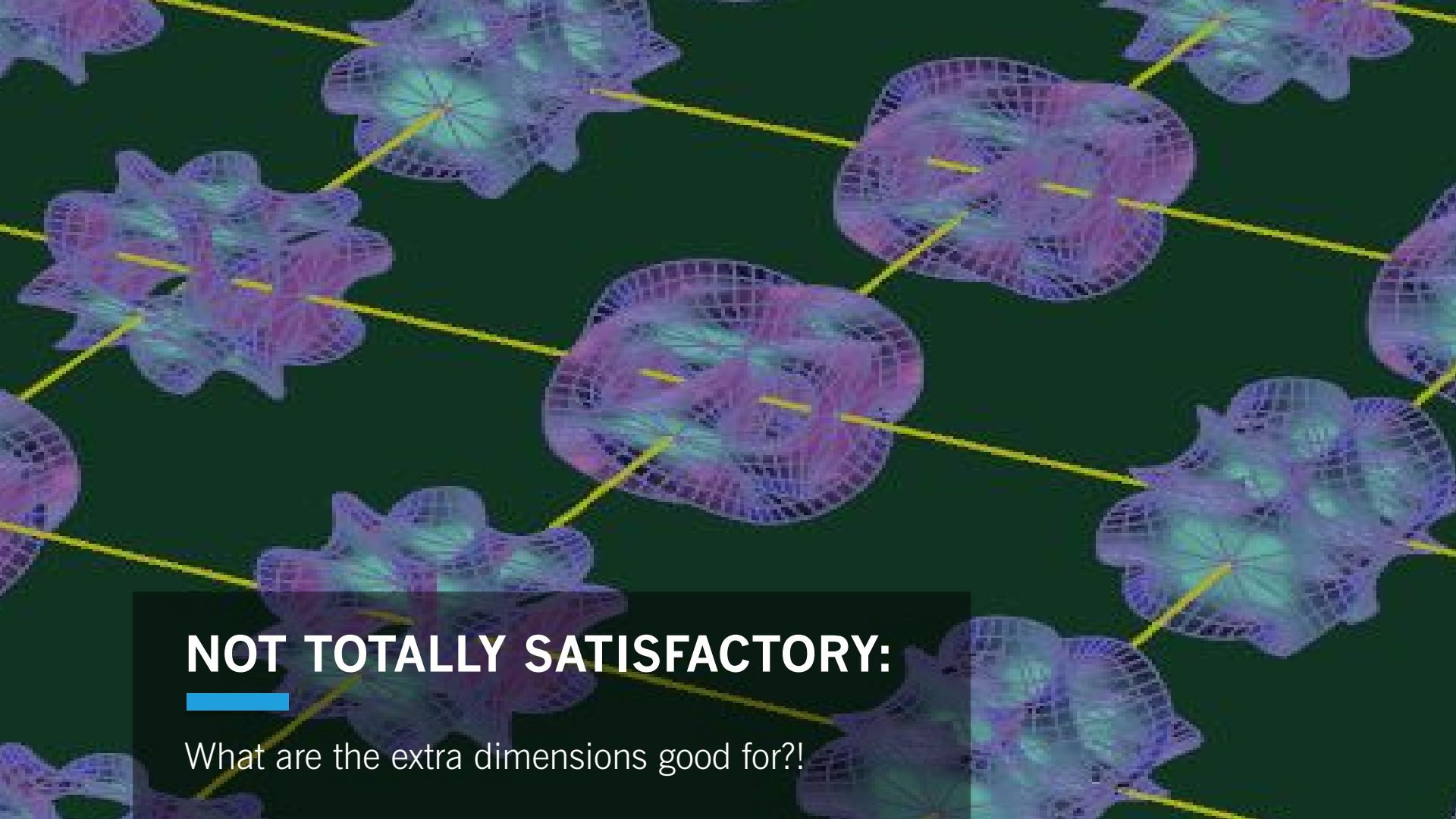
What do we do with the extra 6 dimensions?

Why haven't we seen them if they are around?



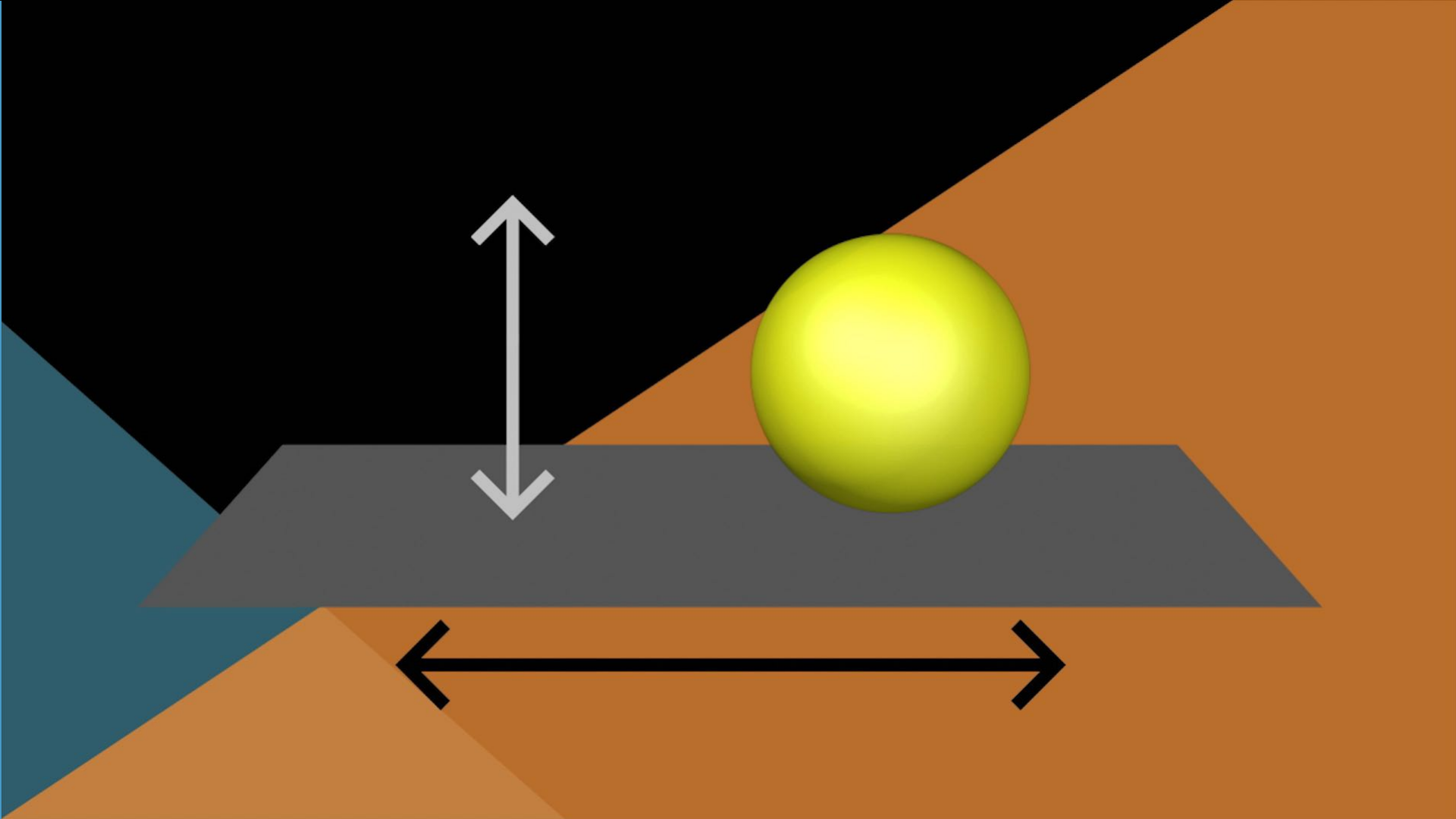
FIRST ATTEMPT AT THE ANSWER:

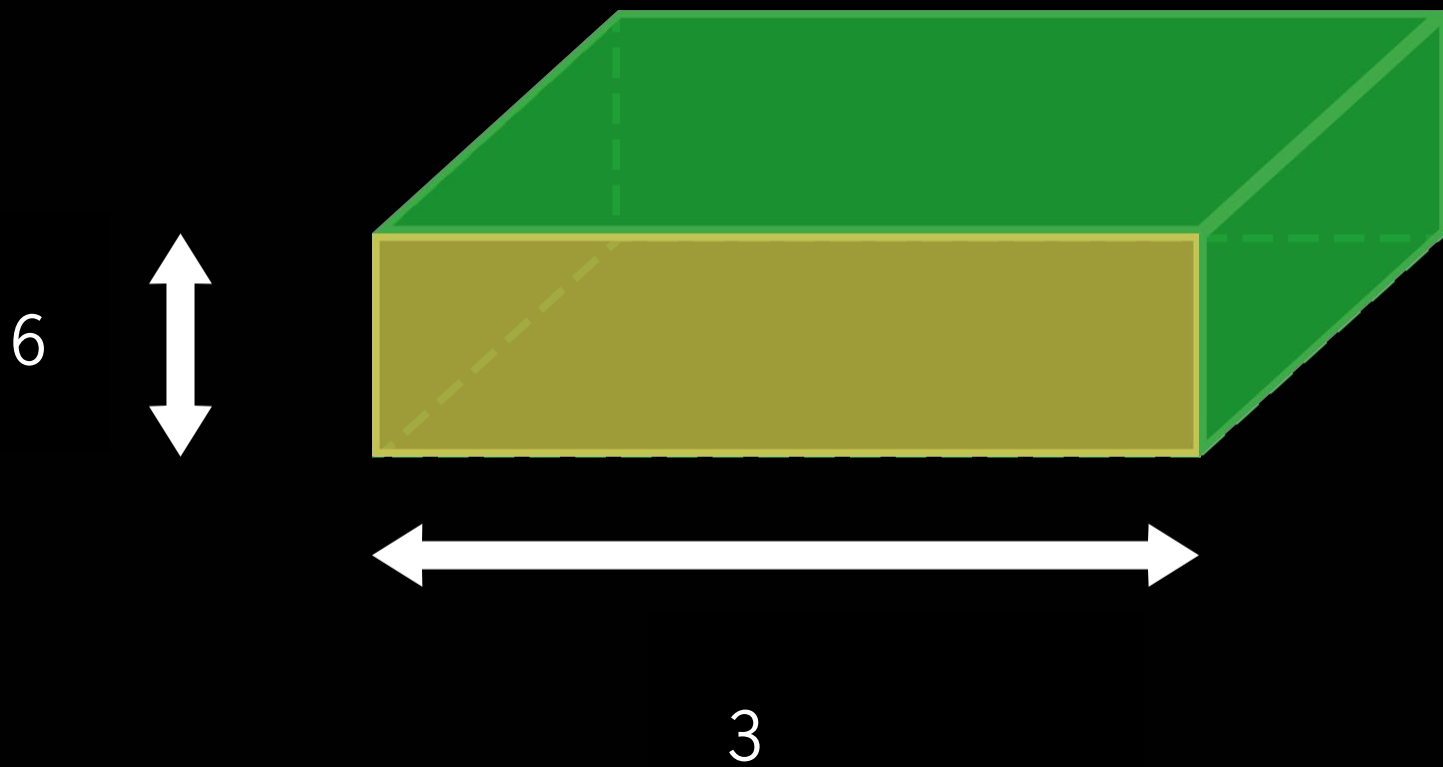
We can demand that the extra dimensions be curled up into a tiny compact space, thus rendering it unobservable and avoiding a direct clash with reality!

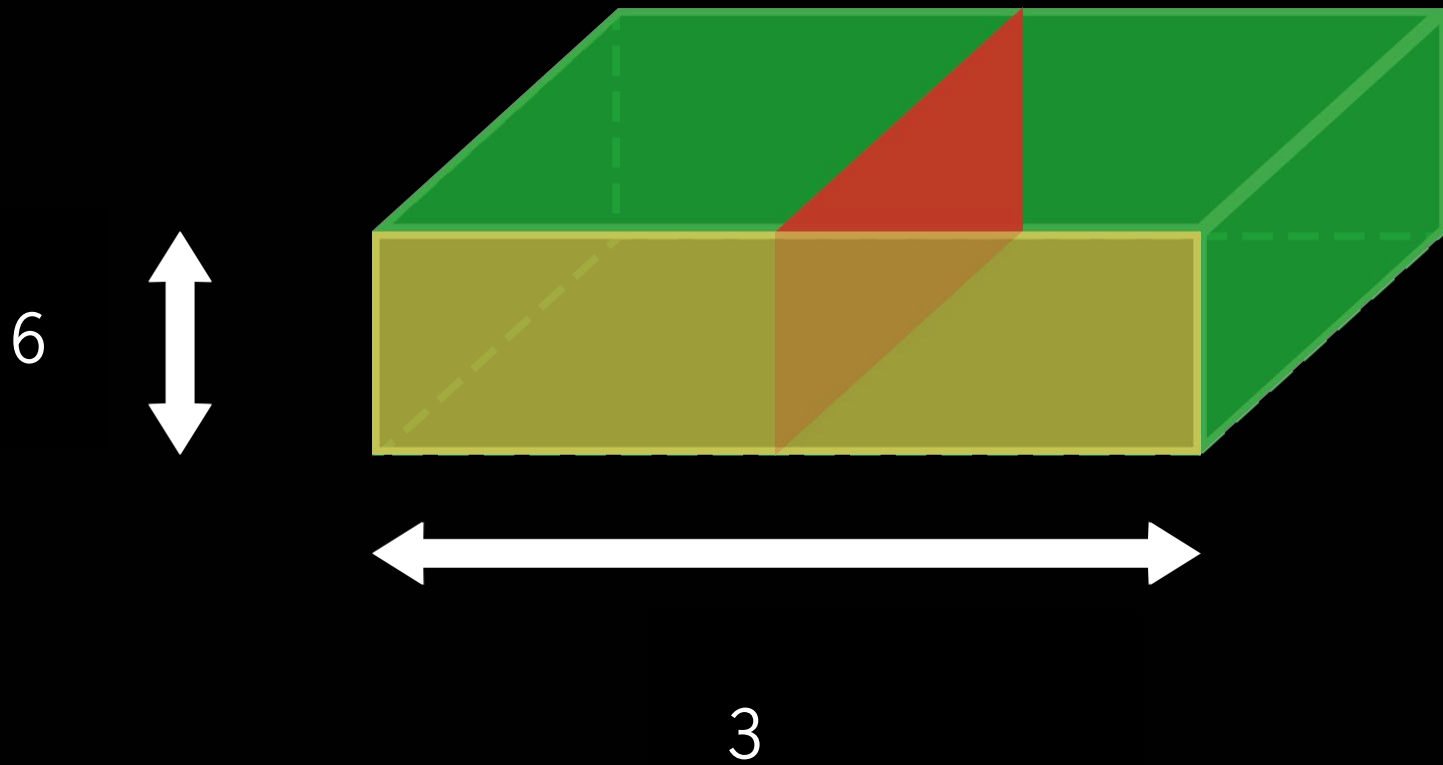


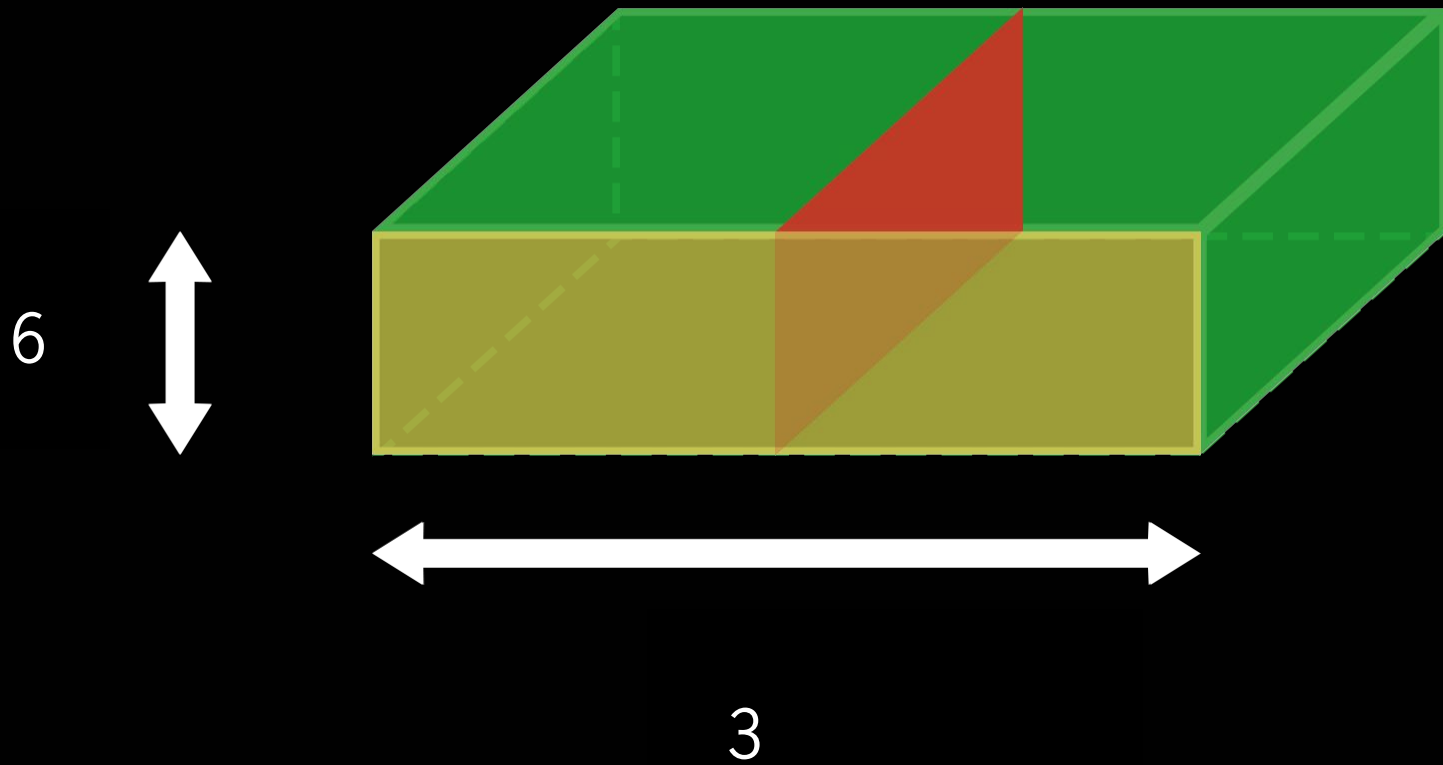
NOT TOTALLY SATISFACTORY:

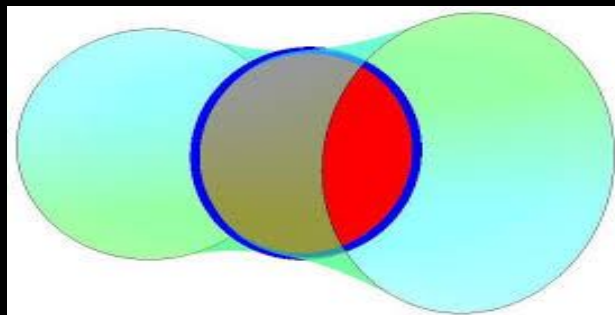
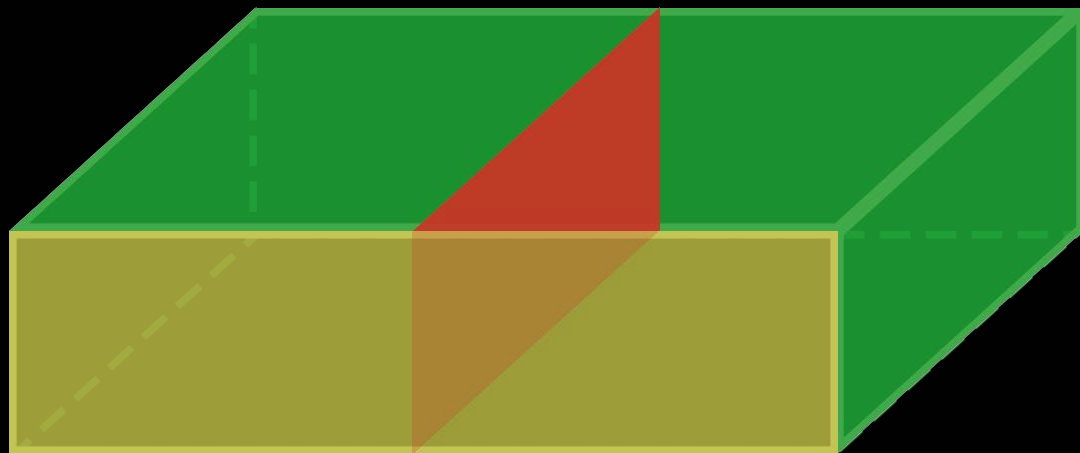
What are the extra dimensions good for?!

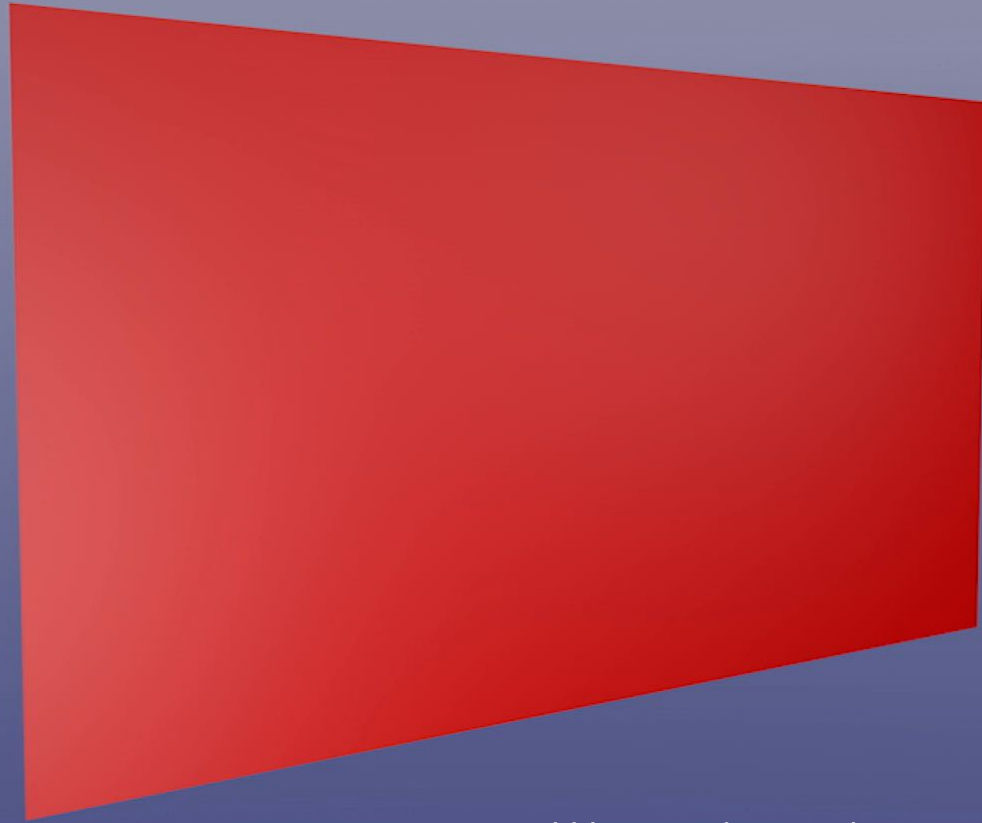












Wrapped membrane can behave like a lower-dimensional object, such as a string

THE THREE STORIES MEET!

RECALL STORY I: Ramanujan was counting partitions of integer: $\exp(c\sqrt{n})$; later interpreted in terms of counting string harmonics.

RECALL STORY II: Hawking found black holes have temperature and entropy. The number of expected states is $N = \exp(A/4)$
He did not know where these states are hiding!

RECALL STORY III: String theory has emerged as a consistent theory of quantum gravity. It however has extra dimensions. What are they good for?

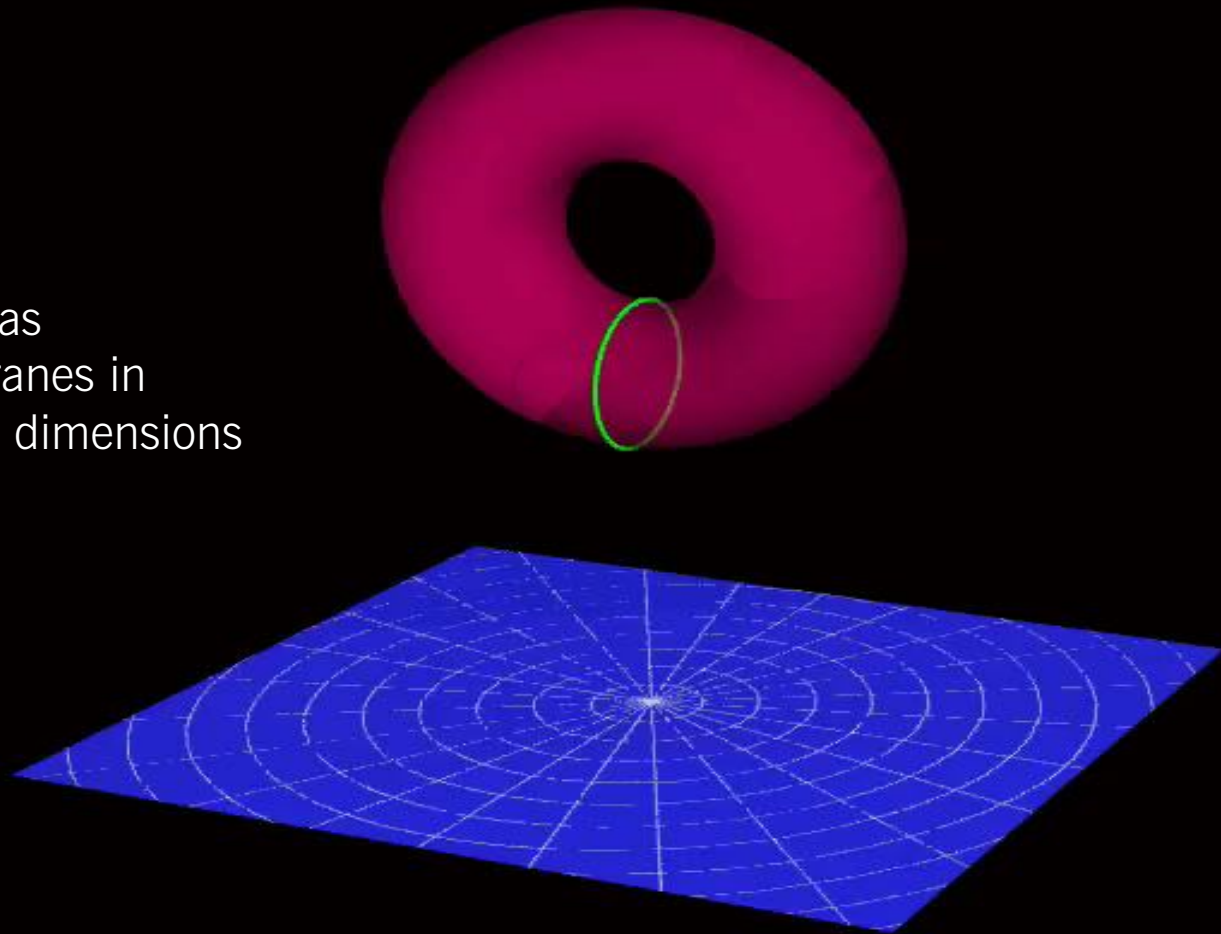
BASIC IDEA



(Joint work with Andy Strominger, 1996):

Use wrapped membranes in string theory to construct black holes, and count how many of them there are and try to check if their number agrees with what Hawking predicted.

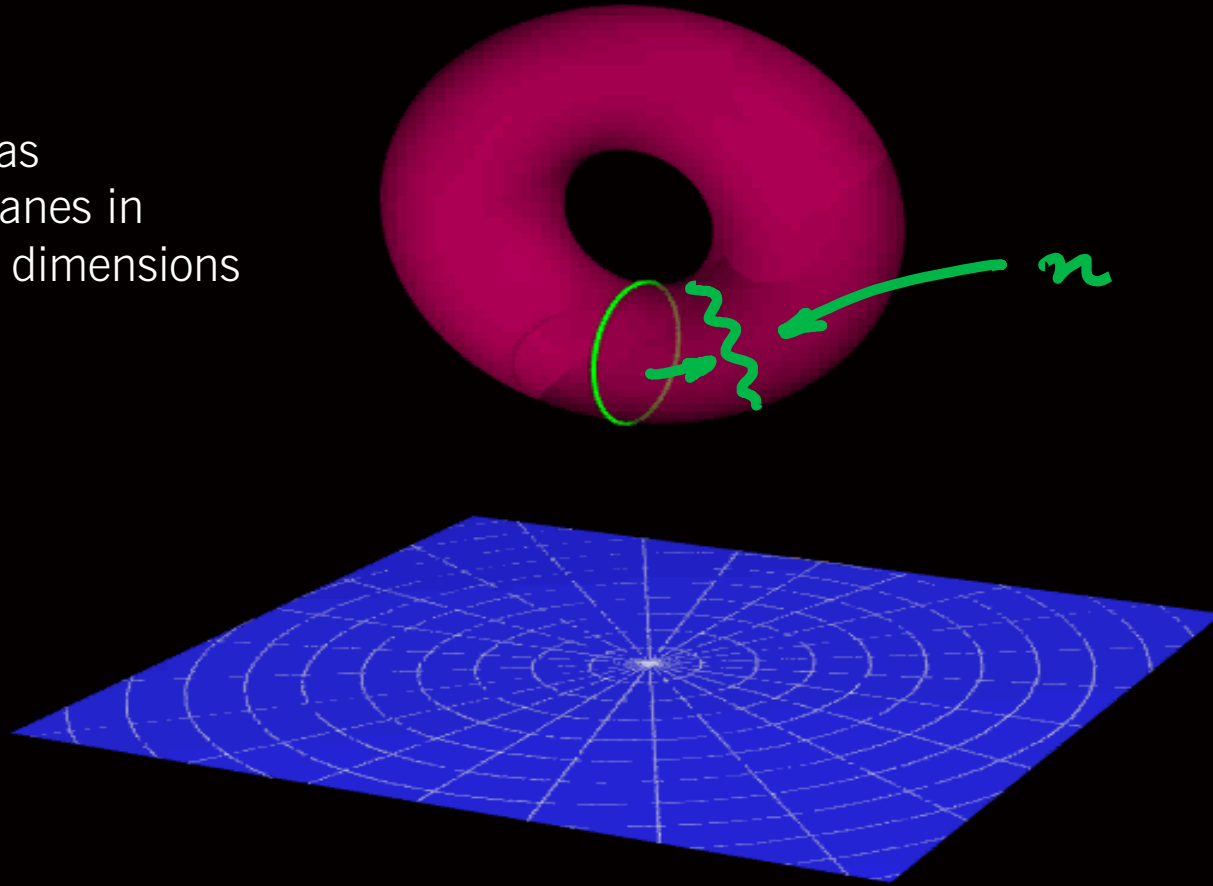
Black hole as
wrapped branes in
the internal dimensions



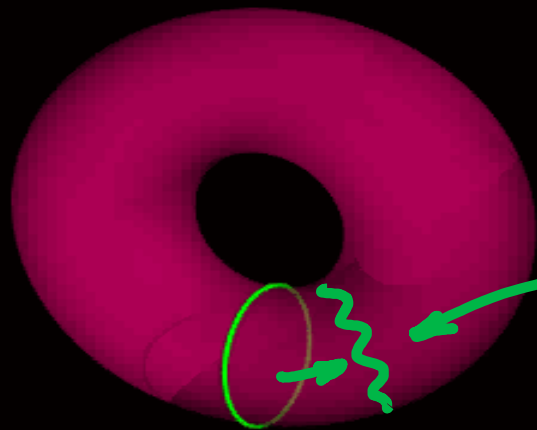
More precisely what was found (in one version of constructing the black hole) is that black holes can be modeled by three-dimensional sheets wrapped around two-dimensional spaces leaving behind a string:

$$3=2+1$$

Black hole as
wrapped branes in
the internal dimensions

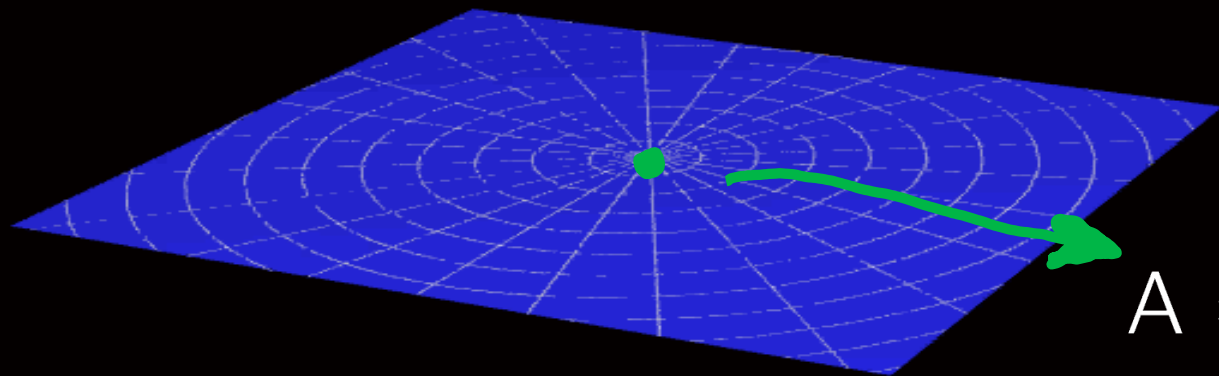


Black hole as
wrapped branes in
the internal dimensions



n

$$N = \exp(c\sqrt{n})$$



[HR]

$$A = 4c\sqrt{n}$$

$$N = \exp(A/4)[H]$$

Thus there is an elegant answer to where the degrees of freedom of the black hole, that Hawking was looking for are hiding:

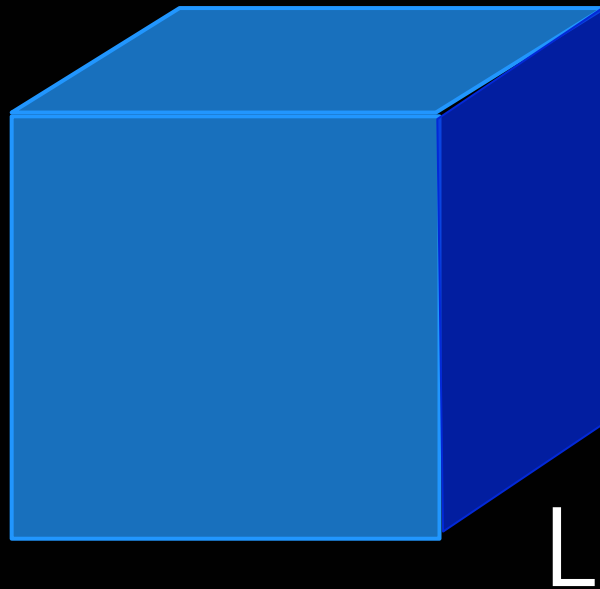
Thus there is an elegant answer to where the degrees of freedom of the black hole, that Hawking was looking for are hiding:

They are hiding in the internal dimensions of string theory.

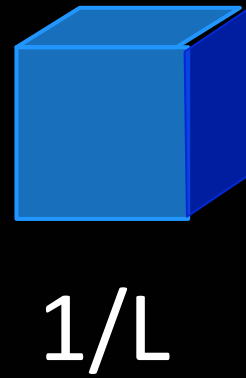
String theory has led to a deep connection between mathematics and physics. The black hole story and the work of Ramanujan and Hardy are just one example of this.

Let us give a few more examples of this deep connection between math and physics in the context of string theory.

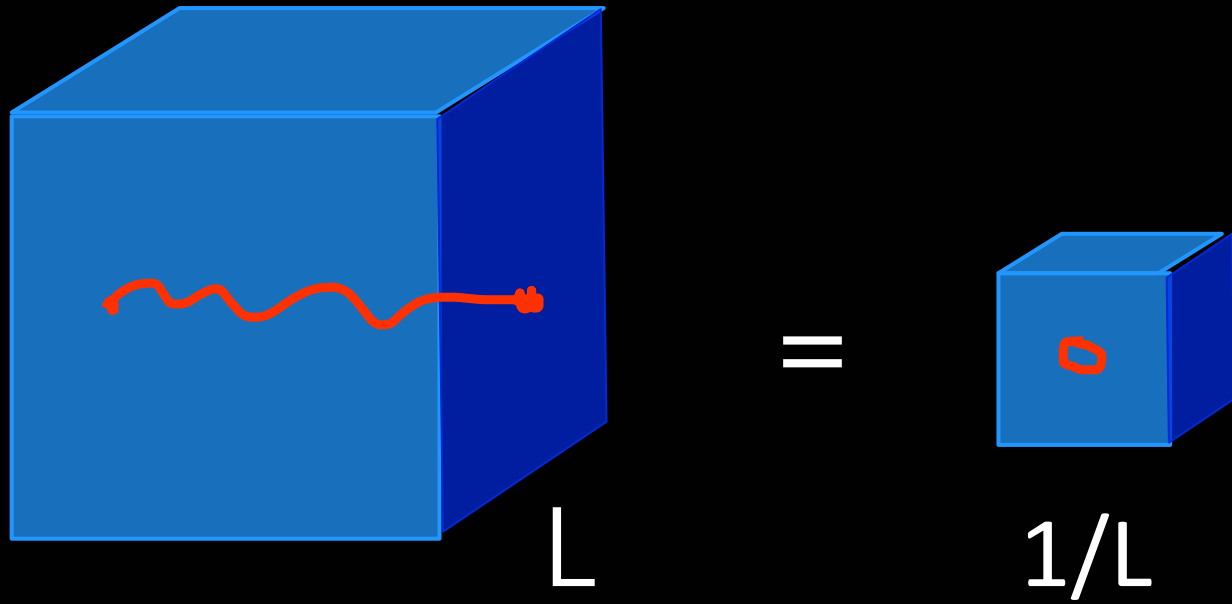
Length inversion symmetry:



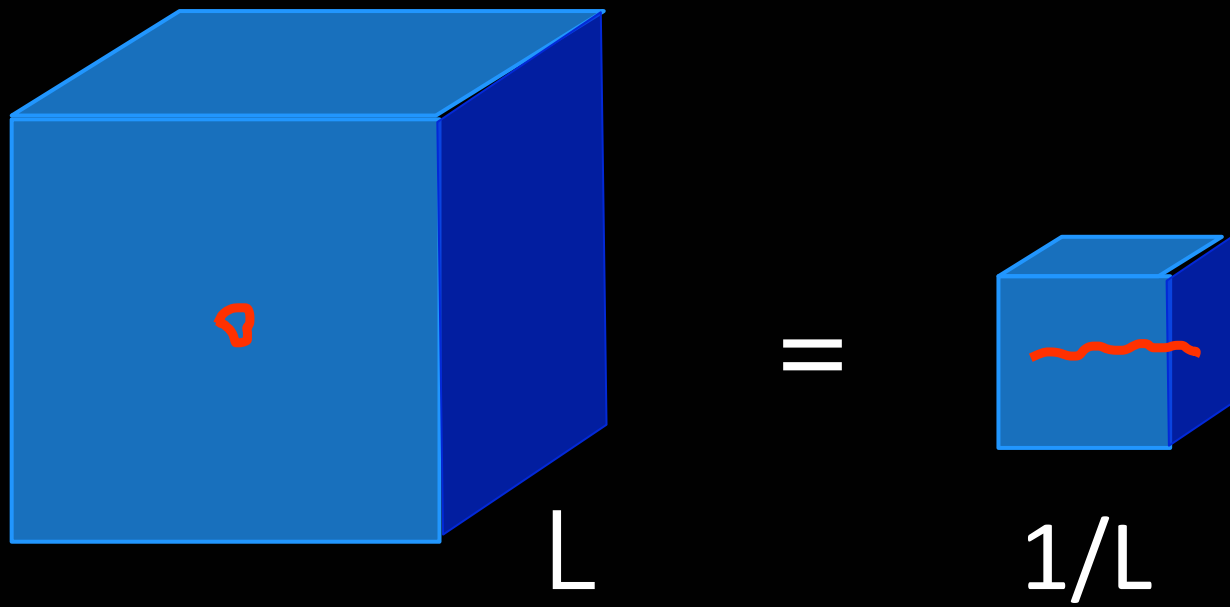
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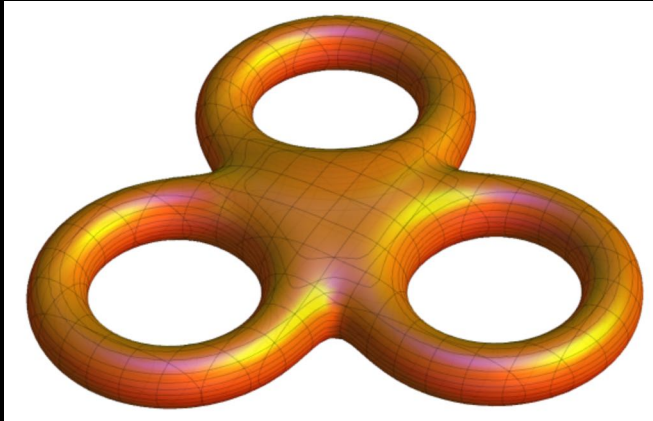
Length inversion symmetry:



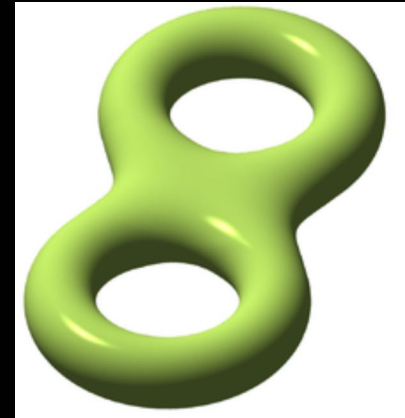
Length inversion symmetry:



Mirror Symmetry: Sometimes spaces with different topologies are equivalent!



=



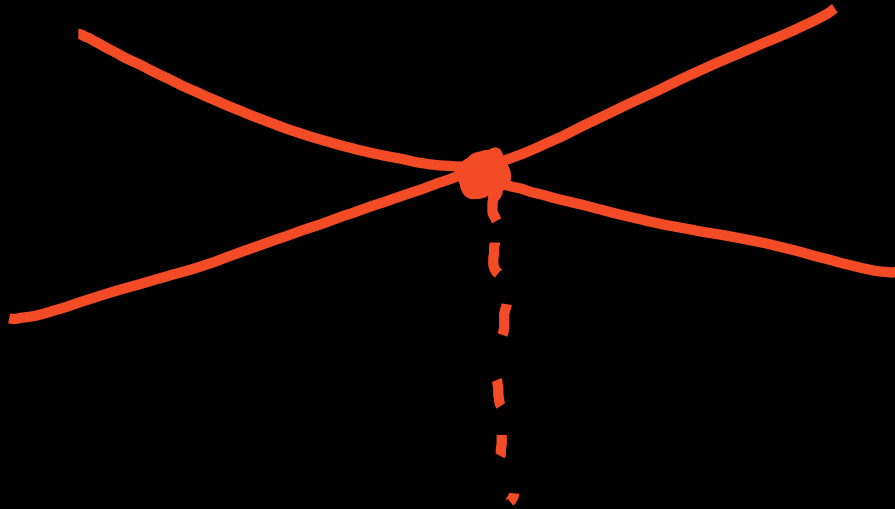
Dualities: Sometimes theories in different dimensions are equivalent!

Example:

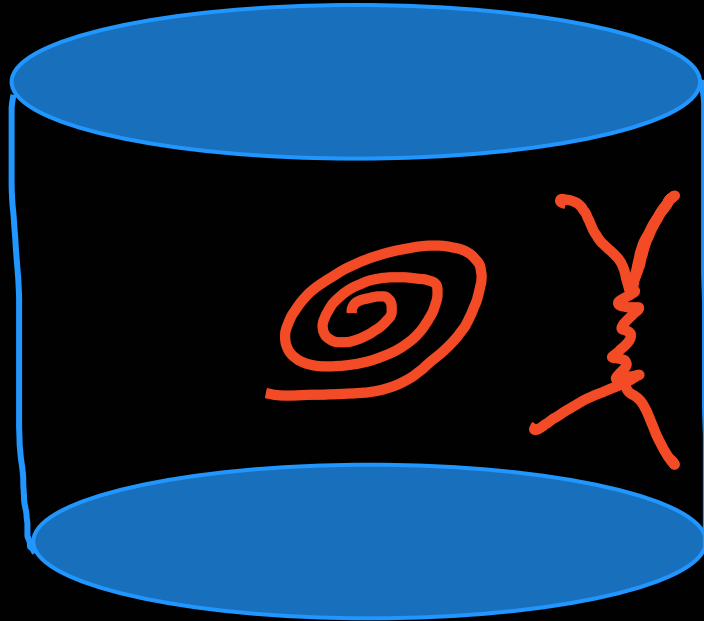
The 11-dimensional M-theory on a 2-torus, when you shrink the area to zero size is equivalent to a 10-dimensional theory (IIB strings) and not a 9-dimensional theory!

$$11 - 2 = 10 !$$

Singularity of space leads to new
physical matter localized at singular loci:



Holography: Electric type forces on the boundary of space can mimic gravity inside



This deep link between physics
and mathematics is sure to
continue for the foreseeable future!

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